METHOD OF EMBANKMENT MODELING USING ONE-DIMENSIONAL LAYERED FINITE ELEMENTS

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Summary:
An original method of computer modeling of ground layers in road embankments has been presented in the paper. A particular areas of presented method application are structures with a large distinctions in the stiffness parameters of each layer of the ground. These changes could be done by an emergency situations or simply by the errors made during the realization processes. The issue of the method is the use of a special finite elements, as a springs, for modeling of whole layered cross-section of road embankment. One single special linear finite element, which describes spring in Finite Element Method, is divided into sub-areas. Each sub-area can be separately integrated and the same can have different stiffness and geometric parameters corresponding to the particular layer of subsoil or road embankment. Shape functions in so defined finite element need to be modified. Modification is necessary for matching the strain field distribution to the changes of the ground parameters. An example for subsidence calculation of layered road embankment has been done. Results were obtained from the different calculation methods. Comparison of these obtained results has been presented in the paper. A precise description of a special finite element and example of its application in the modeling of road embankments has been explained in the paper.

Keywords: road embankment, subsoil, FEM, shape functions

Introduction

The buildings and the road constructions have their own specific tasks to fulfill. To ensure durability and safety of construction, the issue of cooperation between construction and the subsoil should be considered. The same what is the foundation for building, the same for road construction is subsoil placed exactly under the pavement. If the walls of buildings cracks, it means the foundation is too weak. For comparison, if the pavement surface cracks it might be a sign that embankment is too weak. Therefore in the pavement surface structure could appear damages, as a result of degradation of the embankment layers. In addition, in this case there is another factor causing the greatest danger. This factor is of course water, which penetrates the pavement surface structure for example through the dilatation. The result of it might be a significant weakening of the subsoil capacity. A dank embankment lose capacity due to sedimentation and subsidence under load. In winter, there is another risk in form of uneven lifting of the ground. As a result of it, pavement surface is lifted, as well. In this case the pavement plate could be easily broken under its own weight or under car load.

Therefore there is a problem with a proper modeling and the same with a calculation of road embankments. Especially embankments, which might have design errors or defects made during the realization process. Very often on the construction site a lot of errors and mistakes are made. As a reasons of it could be listed for example remissness or simply lack of knowledge and experience of workers. The consequence of not sufficient realization is lack of proper embankment strength. That is why first of all special attention should paid to the mistakes made during the compacting of embankment layers. The process of ground densification is very important for giving a proper load capacity for each layer. Defects of one road embankment layer can influence of stability and capacity of all construction in this range. So it is very important to control the quality and parameters of each embankment layer during the erection process.

The idea of one-dimensional special finite elements application for Finite Elements Method modeling analysis of layered road embankment, has been presented in the paper. In the analysis, one of the simplest ways of computer modeling of ground, which is the Winkler's model, has been used. In this model the subsoil is considered as one-dimensional set of springs. These springs have assigned appropriate stiffness parameters, to describe the characteristic of the subsoil.

In justified cases, the Winkler's method can be used in modeling multilayered subsoil. In this model, subsoil is considered as a set of springs connected in series, where each layer of springs describes a particular layer of embankment (Fig. 1). So the stiffness parameters of springs are corresponded to adequate layer of subsoil. Eventually resultant stiffness is achieved as a result of the relevant summation of stiffness components. Different idea of the resultant stiffness determination with the direct use of Finite Element Method algorithms (Bathe 1996) (Zienkiewicz et al., 2005) is proposed in the paper.

The conception of special finite elements

In the analysis, model of interaction between concrete plate (pavement) and road embankment based on Winkler's hypothesis and application of one-dimensional springs, as a subsoil model, has been used (Fig. 1b). The idea of presented conception is application of special linear finite elements (springs). All layered cross-section of embankment can be modeled by one single element (Fig. 1c). The conception has been presented in the Figure 1.

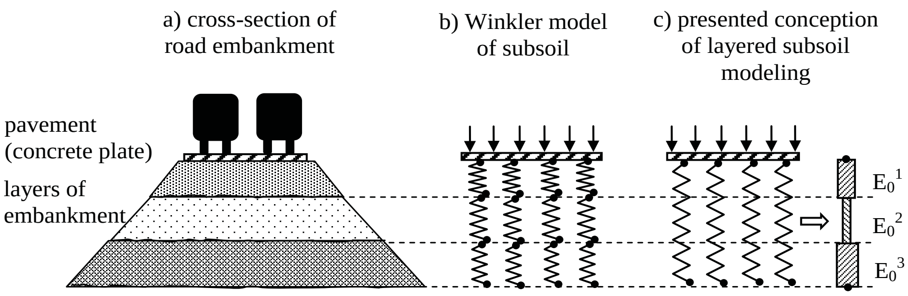


Fig. 1. Conception of one-dimensional layered finite elements in road embankment modeling.

In modeling of layered embankment by one finite element, there is a problem with appropriate description of influence of the individual layer stiffness parameters on the resultant stiffness. The problem is solved by the use of a special finite element (spring) with variable stiffness in the areas of approximated strain field and with the possibility of controlled changes in stiffness and geometry. Structure of defined finite element requires integration in sub-areas. The conception of elements integrated in sub-areas is based on a division of element *e* into *n* parts (sub-areas), Figure 2. The stiffness matrix for one element *e* is obtained by the aggregation of partial stiffness matrix from each sub-area, which is given by:

$$\mathbf{K}_e = \mathbf{K_1} + \mathbf{K_2} + \mathbf{K_3} + \ldots = \int_0^{g_1} \mathbf{K}_e^1(\xi) + \int_{g_1}^{g_2} \mathbf{K}_e^2(\xi) + \int_{g_2}^{g_3} \mathbf{K}_e^3(\xi) + \ldots = \sum_{k=1}^{n} \mathbf{K}_e^k \tag{1}$$

Stiffness matrix for the sub-area *k* of finite element *e* (spring) is determined from the equation:

$$\mathbf{K}_e^k = \oint_{V_k} \mathbf{B}_e^{k\mathrm{T}} \cdot \mathbf{D}_e^k \cdot \mathbf{B}_e^k \cdot \mathrm{dV}_k \tag{2}$$

where: $\mathbf{B}_e^k$ – strain shape matrix of sub-area (layer) *k*,
$\mathbf{D}_e^k$ – elasticity matrix of sub-area (layer) *k*,
V_k – integration volume,
g_k – the depth of the layer *k* bottom.

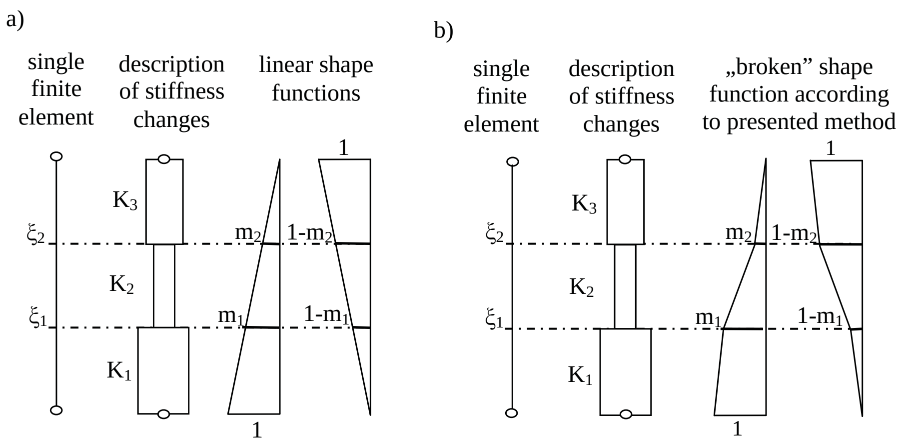


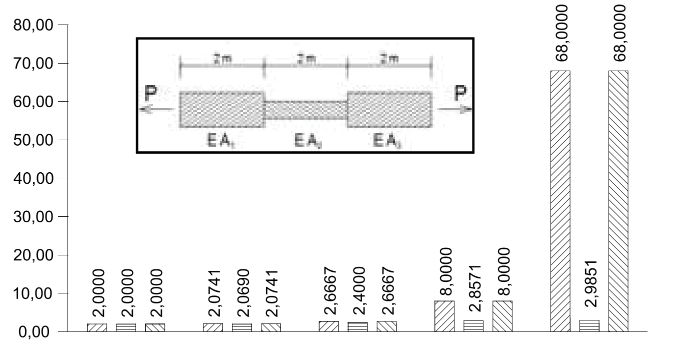
Fig. 2. Division of the finite element into sub-areas.

So defined finite element, shown in Figure 2a, with assumed linear distribution of strain field (standard solution) leads to an erroneous results. It has been illustrated in Figure 3 (Mackiewicz 2012). This is related to the effect of the finite element stiffness increase.

The reason of it, is lack of strain changes adaptability to differences in stiffness parameters inside of the element. To eliminate this effect "broken" shape functions have been used, Figure 2b. Stiffness matrix of so defined finite element is described in the following form:

$$\mathbf{K}_{\mathbf{e}}^{\mathbf{k}} = b^2 \cdot \frac{EA}{L} \cdot \int_{\xi_1}^{\xi_2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} d\xi = \begin{bmatrix} b^2(\xi_2 - \xi_1)\cdot\frac{EA}{L} & -b^2(\xi_2 - \xi_1)\cdot\frac{EA}{L} \\ -b^2(\xi_2 - \xi_1)\cdot\frac{EA}{L} & b^2(\xi_2 - \xi_1)\cdot\frac{EA}{L} \end{bmatrix} \quad \text{gdzie} \quad b = \frac{m_2 - m_1}{\xi_2 - \xi_1} \qquad (3)$$

The replacement linear distribution of the shape functions by the "broken" shape functions leads to correct results, as illustrated in Figure 3.



$A_1 = A_3 = 100\ cm^2$ - stałe	$A_2 = 100\ cm^2$	$A_2 = 90\ cm^2$	$A_2 = 50\ cm^2$	$A_2 = 10\ cm^2$	$A_2 = 1\ cm^2$
Comparative model	2,0000	2,0741	2,6667	8,0000	68,0000
Elements with linear shape function	2,0000	2,0690	2,4000	2,8571	2,9851
Elements with "broken" shape function	2,0000	2,0741	2,6667	8,0000	68,0000

Fig. 3. Comparison of the results obtained from calculations with linear and "broken" shape functions.

The essence of special element conception is the method of estimation of the "broken" shape function parameters. The shape function values m_1 and m_2, in general m_i, matching strain field distribution inside the sub-area to stiffness changes, have been determined from the formulas showed below (Chyzy et al., 1996). These formulas have been obtained from the assumption that the sub-areas create system of springs connected in series. The idea of determination of parameters m_i has been shown in Figure 4.

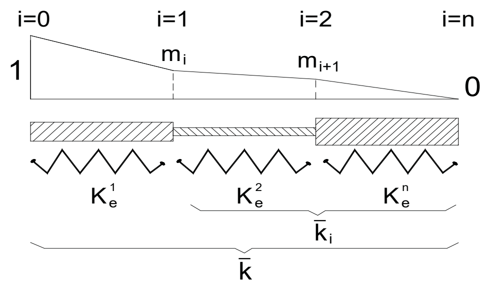


Fig. 4. Determination of parameters m_i.

Adaptation of the formulas for the analysis of layered road embankment is as follows:

$$m_i = \frac{\overline{k}}{\overline{k}_i} \qquad \overline{k} = \frac{1}{\sum_{k=1}^{n} \frac{1}{k_k}} \qquad \overline{k}_i = \frac{1}{\sum_{k=i}^{n} \frac{1}{k_k}}, \quad i = 1,2,..,n-1 \tag{4}$$

where: $\overline{k}$ – resultant stiffness for total spring system,
$\overline{k}$ – resultant stiffness for part of spring system to the point where value m_i is calculated,
$\overline{k}_k$ – stiffness of the *k*-th layer of the embankment,
n – the number of layers. (Mackiewicz 2012).

Another issue related to the embankments modeling with the linear elastic elements (integrated in sub-areas) is adequate determination of the layer stiffness (spring). One of the solution is to use Winkler's hypothesis. According to Winkler's model, the subsidence *s* of the elastic subsoil is proportional to the acting load *q*.

$$q = k_z \cdot s \tag{5}$$

The coefficient of elasticity k_z for a homogeneous subsoil to a depth *z* can be determined according to the formula (Wilun 2005):

$$k_z = \frac{E_0}{\omega \cdot B \cdot \left(1 - \nu^2\right)} \tag{6}$$

The stiffness of the layered subsoil is the sum of the individual layers stiffness. For a single layer *i* stiffness is determined from the equation:

$$k_z^i = \frac{E_0^i}{\Delta\omega_i \cdot B \cdot \left(1 - \nu^2\right)}, \ \Delta\omega_i = \omega_i - \omega_{i-1} \tag{7}$$

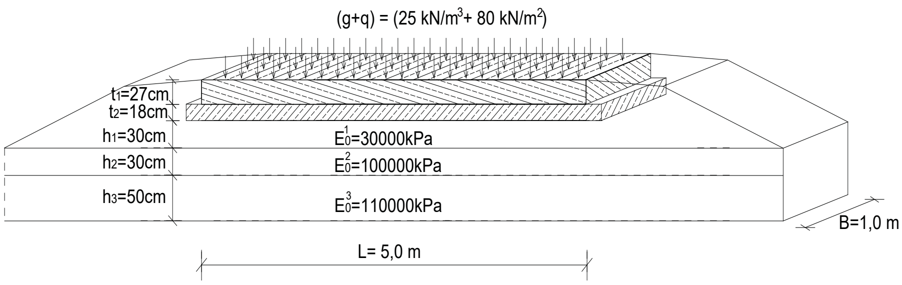
where: *q* – acting load,
E_0 – strain modulus of the subsoil,
B – width of the loaded area,
ν – coefficient of lateral expansion of the subsoil,
ω (ω_i) – influence coefficient, depending on the shape of the loaded area, selected according to appropriate tables and nomograms (Wilun 2005).

Calculation example

In a performed calculation test, the values of the road embankment subsidence were compared. In order to obtain distinction between stiffness of the road embankment layers, different parameters for each layer have been established. Based on known traffic categories and the choice of the substructure material, the following arrangement of pavement and embankment structure has been adopted:

1. The concrete slab, thickness 27cm, concrete B50.
2. The substructure, thickness 18cm, concrete B7,5.
3. The top layer of compacted road embankment, thickness h_1=30cm, strain modulus E_0^1=30000kPa.
4. The middle layer of compacted road embankment, thickness h_2=30 cm, strain modulus E_0^2=100000kPa.
5. The bottom layer of compacted road embankment, thickness h_3=50cm, strain modulus E_0^3=110000kPa.

In the model, dimensions for a concrete slab and substructure with a corresponding thickness, were adopted equal 5,00 m x 1,0 m. The structure consisted of concrete slab and lean concrete substructure was considered as a construction resting on elastic embankment. The analyzed model and the calculation parameters have been shown in Figure 5.



$g = 25$ kN/m^3 – the weight of the concrete,
q=80 kN/m^2 – the load from the cars,
$\nu = 0.3$ – coefficient of lateral expansion of the soil,
$\Delta\omega_1 = 0.156 - 0 = 0.156$ – influence coefficient for first layer,
$\Delta\omega_2 = 0.308 - 0.156 = 0.152$ – influence coefficient for second layer,
$\Delta\omega_3 = 0.518 - 0.308 = 0.210$ – influence coefficient for third layer.
Influence coefficients obtained according to (Wilun 2005).

Fig. 5. Parameters of the example of road embankment calculation.

As a constant loads were taken dead weight of concrete slab and dead weight of concrete substructure with following value:

$$g = 25\,\text{kN/m}^3 \cdot (0{,}27\,\text{m} + 0{,}18\,\text{m}) = 11{,}25\,\text{kN/m}^2$$

As a variable loads were taken loads from car transport as a two axes equal 100kN on each roadway. This load was turned into distributed load, by the dividing these forces by the roadway area.

$$q = \frac{2 \cdot 100\,\text{kN}}{2{,}5\,\text{m} \cdot 1\,\text{m}} = 80\,\text{kN/m}^2$$

Calculations were made in three variants:

1. Variant I – according to the Winkler's hypothesis, using formulas (5), (6), (7) described by (Wilun 2005).

2. Variant II – as a control variant. Flat (two-dimensional) Finite Element Method model was calculated, as shown in Figure 6a. For modeling of subsoil using the flat elements, modulus transformation must be applied to describe susceptibility of subsoil, as in the Winkler's hypothesis:

$$E^i = E_{0^i} \cdot \frac{h_i}{\Delta\omega_i} \tag{8}$$

3. Variant III – using special elements presented in this paper. It means layered, one-dimensional finite elements, integrated in sub-areas, and implemented in the computer construction analysis system named "Orcan" (http://kmb.pb.edu.pl/dydaktyka/tchyzy/orcan.html), Figure 6b;

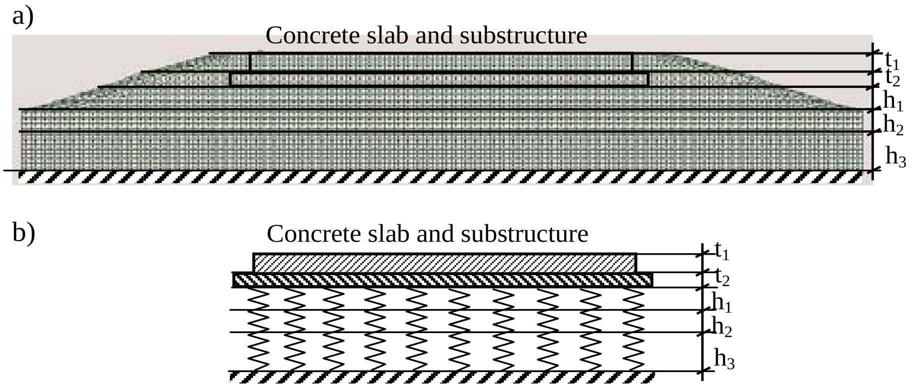


Fig. 6. The computational calculation model: a) two-dimensional Finite Elements Method model with discretisation, b) presented conception of special finite elements.

From the calculations, followed results, as a values of concrete slab subsidence, were obtained:

1. Variant I – the Winkler's hypothesis.

$$s_1 = \frac{q \cdot \Delta\omega_1 \cdot B \cdot \left(1 - v^2\right)}{E_{0^1}} = \frac{91{,}25\text{kN}/m^2 \cdot 0{,}156 \cdot 1{,}0\text{m} \cdot \left(1 - 0{,}3^2\right)}{30000\ \text{kPa}} = 4{,}318 \cdot 10^{-4}\, m = 0{,}4318\ \text{mm}$$

$$s_2 = \frac{q \cdot \Delta\omega_2 \cdot B \cdot \left(1 - v^2\right)}{E_{0^2}} = \frac{91{,}25\text{kN}/m^2 \cdot 0{,}152 \cdot 1{,}0\text{m} \cdot \left(1 - 0{,}3^2\right)}{100000\ \text{kPa}} = 1{,}262 \cdot 10^{-4}\, m = 0{,}1262\ \text{mm}$$

$$s_3 = \frac{q \cdot \Delta\omega_3 \cdot B \cdot \left(1 - v^2\right)}{E_{0^3}} = \frac{91{,}25\text{kN}/m^2 \cdot 0{,}210 \cdot 1{,}0\text{m} \cdot \left(1 - 0{,}3^2\right)}{110000\ \text{kPa}} = 1{,}585 \cdot 10^{-4}\, m = 0{,}1585\text{mm}$$

Subsidence: $s = s_1 + s_2 + s_3 = 7{,}165 \cdot 10^{-5}\, m = 0{,}7165\ mm$

2. Variant II – Finite Elements Method model with flat two-dimensional elements.

– first layer: $E^1 = E_{0^1} \cdot \dfrac{h_1}{\Delta\omega_1}$ $\quad E^1 = 30000\ \text{kPa} \cdot \dfrac{0{,}3\ m}{0{,}156} = 57692{,}31\ \text{kPa}$

– second layer: $E^2 = E_{0^2} \cdot \dfrac{h_2}{\Delta\omega_2}$ $\quad E^2 = 100000\ \text{kPa} \cdot \dfrac{0{,}3\ m}{0{,}152} = 197368{,}42\ \text{kPa}$

– third layer: $E^3 = E_{0^3} \cdot \dfrac{h_3}{\Delta\omega_3}$ $\quad E^3 = 110000\ \text{kPa} \cdot \dfrac{0{,}5\ m}{0{,}210} = 261904{,}76\ \text{kPa}$

Subsidence: $s = 7{,}272 \cdot 10^{-5}\ m = 0{,}7272\ mm$

3. Variant III – special linear elastic finite elements integrated in sub-areas (presented method).

$$k_{s^1} = \frac{E_{0^1}}{\Delta\omega_1 \cdot B \cdot (1 - v^2)} \cdot A = \frac{30000\ \text{kPa}}{0{,}156 \cdot 1\text{m} \cdot (1 - 0{,}3^2)} \cdot (1\text{m} \cdot 0{,}05\text{m}) = 211327{,}134 \frac{\text{kN}}{m^3} \cdot 0{,}05\text{m}^2 = 10566{,}36 \frac{\text{kN}}{m}$$

$$k_{s^2} = \frac{E_{0^2}}{\Delta\omega_2 \cdot B \cdot (1 - v^2)} \cdot A = \frac{100000\ \text{kPa}}{0{,}152 \cdot 1\, m \cdot (1 - 0{,}3^2)} \cdot (1\text{m} \cdot 0{,}05\text{m}) = 722961{,}25 \frac{\text{kN}}{m^3} \cdot 0{,}05\text{m}^2 = 36148{,}06 \frac{\text{kN}}{m}$$

$$k_{s^3} = \frac{E_{0^3}}{\Delta\omega_3 \cdot B \cdot (1 - v^2)} \cdot A = \frac{110000\ \text{kPa}}{0{,}210 \cdot 1\text{m} \cdot (1 - 0{,}3^2)} \cdot (1\text{m} \cdot 0{,}05\text{m}) = 575614{,}86 \frac{\text{kN}}{m^3} \cdot 0{,}05\text{m}^2 = 28780{,}74 \frac{\text{kN}}{m}$$

Resultant stiffness:

$$\bar{k}_s = \frac{1}{\frac{1}{k_{s^1}} + \frac{1}{k_{s^2}} + \frac{1}{k_{s^3}}} = \frac{1}{\frac{1}{10566{,}36 \frac{\text{kN}}{m}} + \frac{1}{36148{,}06 \frac{\text{kN}}{m}} + \frac{1}{28780{,}74 \frac{\text{kN}}{m}}} = 6367{,}42 \frac{\text{kN}}{m}$$

Subsidence: $s = 7{,}290 \cdot 10^{-5}\ m = 0{,}7290\ mm$

Conclusions

Using the presented method, for the calculation of the road embankment with large distinctions in the layer parameters, a sufficiently accurate results were achieved. Simultaneously the minimum number of unknowns were provided – it means regardless of the number of subsoil layers, the number of unknowns is the same (3D elastic elements have two nodes, in each node 3 steps of freedom).Thus calculations according to the method presented in the paper lead to correct results with less number of finite elements. It means that in this case the proposed solution is more effective.

The presented method is also an example of a specific application of known solutions – the Winkler's hypothesis. However the prospective application of the presented method is post-critical analysis of the constructions with a large stiffness changes, what is still developed by the authors (Chyzy 2009). Expected advantage of the special finite elements is also the elimination of the necessity of repeated rearrangement algorithms for Finite Elements Method mesh discretisation. The necessity of computationally expensive discretisation rearrangement could be caused by sudden changes in the parameters of the embankment. Definitely the presented approach can be used in stationary solutions with abrupt and step changes of stiffness, what has been presented in the paper.

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