# THE ULTIMATE FATIGUE STATE OF RC STRUCTURES ACCORDING TO MODEL CODE 2010

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**Summary:**
The paper presents a review and discussion of CEB-FIP Model Code 2010 principles and rules of design proposed for the needs of verification of fatigue strength of RC structures, like concrete bridges, subjected to dynamic actions. It has been stressed that these rules are based on some different assumptions compared to Eurocode 2. The paper presents also an example of calculations performed to indicate the differences between the results of verification of the Ultimate Fatigue State according to the Model Code 2010, and the the Eurocode 2 and former Polish Codes. The differences are shown in relevant tables.

**Keywords:** Ultimate Fatigue State, RC structures, Cyclic loads. Model Code 2010

## Introduction

The first complete draft of CEB-FIP Model Code 2010 was approved in October 2011 and the final version was published in 2012. This document has been elaborated by an international team of researches and experts aimed to define new and advanced solutions and directions in designing of reinforced concrete structures. It is characterized by different attitude to verifying the Ultimate Fatigue State (UFS) of RC structures for both concrete and reinforcement.

The aim of this paper is to present principles and rules for verification of the UFS of RC structures in accordance to CEB-FIP Model Code 2010. Additionally, example calculations were performed, which were the base for the comparison of CEB-FIP Model Code 2010 and former Polish Codes for bridges and other engineering structures discussed in some papers (Woliński, 1998, 1999); (Jankowiak and Madaj, 2011); (Siwowski and Michalak, 2013).

This paper was elaborated in the frame of state of art analysis of the dynamic fatigue behavior RC structural members made of recycled aggregate concrete subjected to cyclic load (including low-cycle fatigue). That problem is till now at the pilot phase and only very few papers are presented, e.g. (Gordon, 2011) or (Luo and Yao, 2011a, 2011b).

## Methods of verifying of RC structures fatigue strength proposed in Model Code 2010

Model Code 2010 (MC 2010, 2012) includes in one document principles and rules for design RC buildings, as well as bridges and other engineering structures throughout their all the life cycles, beginning from designing to erecting, maintenance, strengthening and finally demolition.

In the section describing the ULS state for non-static loads, the rules for shaping structural elements due to fatigue strength have been enclosed. There are also presented a restrictions for reinforcing and prestressing steel as calculation procedures which can be applied for cyclic loads with number of at least $10^4$ repeats. It should be mentioned however that the low-cycle fatigue is not enclosed in Model Code 2010 design procedures. Just like in current Eurocode 2 (PN-EN 1992-1-1, 2008), the verification of the UFS include separate courses for steel and concrete, where both conditions should be fulfilled. The Palmgren-Miner summation (Palmgren, 1945) or adequate S-N curves (Fig. 1), different than in PN-EN 1992, also have been applied here.

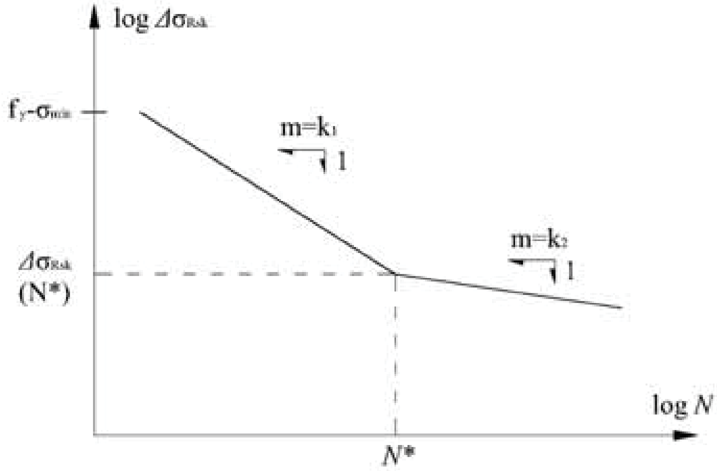


Fig. 1. Characteristic S-N curves for reinforcing and prestressing steel based on the CEB-FIP Model Code 2010 (described in Table 1 and 2)

Model Code 2010 presents 3 main methods with varied levels of accuracy.

- **Simplified procedure** can be used only in case of objects subjected to the number of loading cycles not higher than $10^8$, with low stress range.

The reinforcement fatigue state requirements will be met if:

$$\gamma_{Ed} \cdot \max \Delta\sigma_{Ss} \leq \frac{\Delta\sigma_{Rsk}}{\gamma_{s,fat}} \tag{1}$$

where $\max \Delta\sigma_{Ss}$ is the maximum steel stress range under acting loads, $\Delta\sigma_{Rsk}$ is a characteristic value of fatigue strength under the number of $10^8$ cycles given in Table 1 and 2, dependent on reinforcement type, its diameter, shape, environment, type of bond. Adopted value of load coefficient $\gamma_{Ed}$ is 1,1, but in case if stress analysis is accurate enough and its results are confirmed with in-situ observation - $\gamma_{Ed} = 1,0$ can be adopted. Another parameter is steel fatigue coefficient $\gamma_{s,fat} = 1,15$ just like in Eurocode 2.

Tab. 1. S-N parameters for reinforcing steel in accordance to CEB-FIP Model Code 2010

| Reinforcement type | $N*$ | Stress exponent | | $\Delta\sigma_{Rsk}$ **(MPa)**[e] | |
|---|---|---|---|---|---|
| | | $k_1$ | $k_2$ | **at $N*$ cycles** | **at $10^8$ cycles** |
| Straight and bent bars $D \geq 25\varnothing$<br>$\varnothing \leq 16mm$<br>$\varnothing > 16mm^{(a)}$ | $10^6$<br>$10^6$ | 5<br>5 | 9<br>9 | 210<br>160 | 125<br>95 |
| Bent bars $D < 25\varnothing^{(b)}$ | $10^6$ | 5 | 9 | $\_^{(c)}$ | $\_^{(c)}$ |
| Welded bars [b] including tack Welding and butt joints Mechanical connectors | $10^7$ | 3 | 5 | 50 | 30 |
| Marine environment [b],[d] | $10^7$ | 3 | 5 | 65 | 40 |

The values $\Delta\sigma_{Rsk}$ represent the S-N curve of a 40 mm bar; for diameters between 16 and 40 mm interpolation between the values of this line and those of line above is permitted.
Most of these S-N curves intersect the curve of the corresponding straight bar. In such cases the fatigue strength of the straight bar is valid for cycle numbers less than that of the intersection point.
Values are those of according straight bar multiplied by a reduction factor depending on the ratio of the diameter of mandrel $D$ and bar diameter $\varnothing$: $\xi = 0,35 + 0,026 \cdot D/\varnothing$.
Valid for all ratios $D/\varnothing$ and diameters $\varnothing$.
In cases where $\Delta\sigma_{Rsk}$ - values calculated from the S-N curve exceed the stress range $f_y - \sigma_{min}$, the value $f_y - \sigma_{min}$ is valid.

Tab. 2. S-N parameters for prestressing steel in accordance to CEB-FIP Model Code 2010

| Structure type | $N*$ | Stress exponent | | $\Delta\sigma_{Rsk}$ (MPa)[b] | |
|---|---|---|---|---|---|
| | | $k_1$ | $k_2$ | at $N*$ cycles | at $10^8$ cycles |
| Pretensioning | $10^6$ | 5 | 9 | 160 | 95 |
| Posttensioning<br>Curved tendons<br>Straight tendons<br>Mechanical connectors | <br>$10^6$<br>$10^6$<br>$10^6$ | <br>3<br>5<br>3 | <br>7<br>9<br>5 | <br>120<br>160<br>80 | <br>65<br>95<br>30 |

In case where the S-N curve intersects that of the straight bar, the fatigue strength of the straight bar is valid.
In cases where $\Delta\sigma_{Rsk}$ - value calculated from S-N curve exceeds the $f_y - \sigma_{min}$ stress range, the value $f_y - \sigma_{min}$ is valid.

The reinforcement fatigue state is verified by the use of two conditions – appropriate to compression and tension:

$$\gamma_{Ed} \cdot \sigma_{c,\max} \cdot \eta_c \leq 0,45 \cdot f_{cd,fat} \tag{2}$$

$$\gamma_{Ed} \cdot \sigma_{ct,\max} \leq 0,33 \cdot f_{ctd,fat} \tag{3}$$

where $\sigma_{c,\max}$ and $\sigma_{ct,\max}$ are maximum concrete stress values caused by load combination under compression and tension accordingly, $f_{cd,fat}$ and $f_{ctd,fat}$ are concrete design fatigue tensile and compressive strengths, while $\eta_c$ is the averaging factor considering the stress gradient.

Mentioned $\eta_c$ coefficient can be calculated from equation below:

$$\eta_c = \frac{1}{1,5 - 0,5 \cdot \frac{|\sigma_{c1}|}{|\sigma_{c2}|}} \tag{4}$$

where $|\sigma_{c1}|$ and $|\sigma_{c2}|$ are accordingly the lower and the larger absolute values of the compressive stress within a distance of 300mm from the surface under the same load combination (Fig. 2).

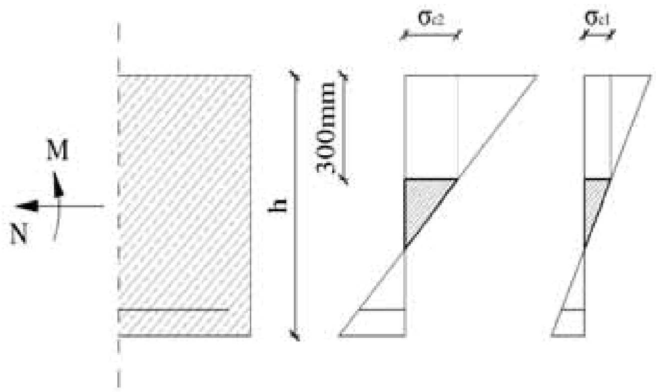


Fig. 2. Definition of the concrete $|\sigma_{c1}|$ and $|\sigma_{c2}|$ in accordance to CEB-FIP Model upper and lower value of compressive stress in Code 2010

The design fatigue reference strength of concrete under tension $f_{ctd,fat}$ is defined as 5-percentage of characteristic concrete tensile strength $f_{ctk,0,05}$ with the use of concrete material factor $\gamma_{c,fat} = 1,50$ as in equation below:

$$f_{ctd,fat} = \frac{f_{ctk,0,05}}{\gamma_{c,fat}} \tag{5}$$

The design fatigue reference strength of concrete under compression is defined taking into consideration: concrete reference strength $f_{cko} = 10MPa$, characteristic compressive strength $f_{ck}$ and the $\beta_{cc}(t)$ coefficient which depends on the age of the concrete in days when the fatigue loading starts, what is shown, similarly to Eurocode 2, in equation:

$$f_{cd,fat} = \frac{0,85 \cdot \beta_{cc}(t)}{\gamma_{c,fat}} \cdot \left[ f_{ck} \cdot \left( 1 - \frac{f_{ck}}{25 \cdot f_{cko}} \right) \right] \tag{6}$$

$\beta_{cc}(t)$ factor is a function of development of concrete compressive strength in time (Eq.7), dependent on the age of concrete (in days) and factor $s$ related to the class of cement used, which values are given in Table 3.

$$\beta_{cc}(t) \aleph \exp\left\{ s \left[ 1 \quad \left( \frac{28}{t} \right)^{ü} \right] \right\} \tag{7}$$

Tab. 3. Values of $s$ factor depending on cement class according to Model Code 2010

| Cement class | 32,5 N | 32,5 R<br>42,5 N | 42,5 R<br>52,5 N<br>52,5 R |
|---|---|---|---|
| $s$ | 0,38 | 0,25 | 0,20 |

- The verification taking into consideration a single, constant range of stress velocity consists in relating the foreseen number *n* of cycles during the use of the structure to maximum fatigue effects *Q*, which, if estimated accurately enough, allows to perform more precise calculations than with the use of the simplified procedure. It is an innovation in comparison with polish codes.

In accordance to reinforcement steel, meeting the criteria is determined by following condition:

$$\gamma_{Ed} \cdot \max \Delta\sigma_{Ss} \leq \frac{\Delta\sigma_{Rsk}(n)}{\gamma_{s,fat}} \tag{8}$$

where $\Delta\sigma_{Rsk}(n)$ is the stress range obtained from a characteristic fatigue strength S-N function, relevant to *n* cycles of variable loading.

The parameters of the S-N curve are shown in Table 1 and 2 and they depend on the reinforcement type, its shape, environment and bonds. Moreover, these are characteristic values, so material safety coefficients are not taken into consideration. Their accurate values should be authorized in proper approvals.

In case of concrete, there is a need to establish the conditions for centric compression, tension and both mixed together, where the verification of fatigue strength state is more indirect than in case of steel. Foreseen number of variable load *n* should be lower or equal to the limit number of cycles *N* that steel or concrete can bear. It is calculated with the use of logarithmic function. For axial compression, the following set of equations is used:

$$\log N_1 = \left(12 + 16 \cdot S_{cd,\min} + 8 \cdot S_{cd,\min}^2\right) \cdot \left(1 - S_{cd,\max}\right) \tag{9}$$

$$\log N_2 = 0,2 \cdot \log N_1 \cdot \left(\log N_1 - 1\right) \tag{10}$$

$$\log N_3 = \frac{\log N_2 \cdot \left(0,3 - 0,375 \cdot S_{cd,\min}\right)}{\Delta S_{cd}} \tag{11}$$

Defining the adequate value of *N* requires fixing the equations above into following conditions:

if $\log N_1 \leq 6$ , then $\log N = \log N_1$ should be assumed (12)

if $\log N_1 \leq 6$ and $\Delta S_{cd} \geq 0,3 - 0,375 \cdot S_{cd,\min}$ , then $\log N = \log N_2$ (13)

if $\log N_1 \leq 6$ and $\Delta S_{cd} < 0,3 - 0,375 \cdot S_{cd,\min}$ , then $\log N = \log N_3$ (14)

and the values $S_{cd,\min}$ , $S_{cd,\max}$ of compressive stress and $\Delta S_{cd}$ compressive stress range nan be calculated from Eq.15:

$$S_{cd,\min} = \frac{\left|\gamma_{Ed} \cdot \sigma_{c,\min} \cdot \eta_c\right|}{f_{cd,fat}} \tag{15}$$

$$S_{cd,\max} = \frac{\left|\gamma_{Ed} \cdot \sigma_{c,\max} \cdot \eta_c\right|}{f_{cd,fat}} \tag{16}$$

$$\Delta S_{cd} = S_{cd,\max} - S_{cd,\min} \tag{17}$$

where $\sigma_{c,\min}$ is the minimum stress under the load combination.

It is essential to mention, that equations above refer to a case when $S_{cd,\min}$ is 0,0÷0,8. However, if $S_{cd,\min}$ is equal or above 0,8, the S-N curves should be used. The environmental conditions are also important. The calculations of $N$ refer to elements working in sealed conditions or large sections of concrete characterized with low permeability. Slim elements which are allowed to dry can reach even higher fatigue strength. On the other hand, concrete structures, which are constantly under the water, can prove lower fatigue strength.

Verification of the ultimate fatigue state for centric and eccentric tension ($\sigma_{ct,\max} > 0,026 \cdot |\sigma_{c,\max}|$) requires only calculation below:

$$\log N = 12 \cdot \left(1 - S_{td,\max}\right) \tag{18}$$

where the maximum value of tensile stress $S_{td,\max}$ is considered as:

$$S_{ctd,\max} = \frac{\gamma_{Ed} \cdot \sigma_{ct,\max}}{f_{ctd,fat}} \tag{19}$$

In the case of compression with tension at the same time ($\sigma_{ct,\max} \leq 0,026 \cdot |\sigma_{c,\max}|$), the verification is the same as before:

$$\log N = 9 \cdot \left(1 - S_{cd,\max}\right) \tag{20}$$

- The verification considering various stress ranges bases on the Palmgren-Miner summation and it is presented in the Model Code 2010 as most accurate. Similarly as in the Eurocode 2, single damages caused by *j* stress ranges are added altogether. According to that, structural fatigue damage D is following:

$$D = \sum_{i=1}^{j} \frac{n_{Si}}{N_{Ri}} \tag{21}$$

where $n_{Si}$ is a number of acting load cycles connected with stress range in steel and current stress range in concrete. $N_{Ri}$ describes the limit number of cycles leading to damage of the structure. In accordance to steel, the limit number of cycles is defined minding increased stress range $\gamma_{Ed} \cdot \gamma_{s,fat} \cdot \Delta\sigma_{Esi}$. $N_{Ri}$ for concrete is calculated with methods mentioned above and it does not require any additional increasing. The use of proper calculating methods (for ex. rain flow) allows to reach the limit fatigue damage $D_{\lim} = 1,0$.

## The example of ultimate fatigue state verification

Assumptions common to all variants of example are shown below.

The calculations were performed for a RC bridge slab, subjected to a multiple variable load with constant amplitude (number of cycles = $10^6$). Design assumptions are listed in Table 4. They were chosen and calculated in such way, that all of the ultimate states (except for fatigue) were fulfilled with a reserve not higher than 10% (according to PN-EN). Such action gave the opportunity to show the need of verification of the ultimate fatigue state in structures under the dynamic load. In order to compare calculating procedures with outdated Polish Codes and the Eurocode, the same set of characteristic stresses in steel and concrete was assumed (in the span section), what is shown in Table 5. The example is made the way that minimum stress comes from constant load, while the maximum stress is caused by both constant and variable loads. Depending on the case, they were multiplied by reliable coefficients.

Tab. 4. Design assumptions used in RC slab example calculations

| | |
|---|---|
| Concrete class | C35/45<br>(PN-EN 206-1:2003+AP1:2004) |
| Steel type | B500SP |
| Cross section $h \times b$ | 900x1000 [mm] |
| Reinforcement sectional area | $A_{s,prov} = 80,40cm^2 \ (10\varnothing 32)$ |
| Concrete cover | $C_{nom} = 45mm$ |
| Effective depth | $d = 819mm$ |
| Span of the slab | $L = 1800mm$ |

The results of all variants calculations with Model Code 2010 are set in table below (Table 5). The example presents that on the base of CEB-FIP proposal, the fatigue requirements may not be met. However, it is noticeable that load-bearing capacity of steel was surpassed by 6%. Such effect is the outcome of wide range of acceptable stress $\Delta\sigma_{Rsk}(n)$ for $10^6$ loading cycles. The final stresses in accordance to PN-EN are 162,5 MPa, while in MC 2010 - 210 MPa.

Tab. 5. Results of calculations of the RC slab example

| Code | Result of steel verification | Result of steel verification |
|---|---|---|
| PN-B-03264:2002 | Ultimate state exceeded in 20% | Ultimate state exceeded in 80% |
| PN-91/S-10042 | Ultimate state filled with reserve of 6% | Non checkable |
| PN-EN 1992-1-1:2005+AC:2008 | Ultimate state exceeded in 28% | $\sigma_{cd,\max} > f_{cd,fat}$ |
| PN-EN 1992-2:2005+AC:2008 | Ultimate state exceeded in 86% | $\sigma_{cd,\max} > f_{cd,fat}$ |
| CEB-FIP Model Code 2010 | Ultimate state exceeded in 9% | $\sigma_{cd,\max} > f_{cd,fat}$ |

Referring to the safety reserves that fatigue compressive strength of concrete $f_{cd,fat}$ include, the Model Code 2010 confirmed a huge discrepancies between the results of resaerch on the concrete fatigue.

**Summary**

The article characterizes requirements of the Model Code 2010 for verifying the ultimate fatigue state of RC structures made of normal concrete, however the structures made of recycling aggregate concrete are not included in this Standard.

The results of calculation on the basis of Model Code 2010 set in Table 6 confirm the remarkable influence of the fatigue on the structure's load-bearing capacity. The Eurocode 2 and Model Code 2010 procedures have a similar attitude to the material fatigue, using the S-N curves, design values of fatigue forces and Palmgren-Miner summation of destructions.

In the Model Code 2010 there are no proper rules or calculating methods for the low-cycle fatigue given in the Model Code 2010 and it still requires a respectable research.

Defining the parameters of S-N curves for reinforcing steel, the MC 2010 proposes a case of marine environment, which significantly lowers durability of the structure.

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