CONFORMITY TESTING OF CONCRETE COMPRESSIVE STRENGTH BASED ON OVERLAPPING GROUPS

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Summary:
Conformity testing carried out according to the standard PN-EN 206-1 concerning compressive strength allows the use of two computational procedures. Standardized criteria are checked on a series of a fixed number, created with subsequent strength measurements. The series may overlap or be non-overlap. This paper presents the analysis of the risks borne by the manufacturer using these computational procedures carried out during the compulsory conformity testing. The risk assessment was carried out by means of the operating characteristic (OC-curve). Probabilities of acceptance have been determined using the Monte Carlo simulation method . The calculations of results have shown that the aggregation of data in the series does not affect the risk of rejection of a „good" concrete batch. The author recommends conformity control based on the results of overlapping sequences due to the possibility of early detection of non-compliance, which gives the opportunity to take corrective action as soon as possible.

Keywords: concrete, conformity criterion, overlapping groups

Introduction

Compressive strength of concrete is one of the basic parameters indicative of its quality. The basis of assessment of concrete quality are tests carried out in the framework of statistical quality control on a series of n strength test results obtained from standard samples. Fulfilling the standard conformity criterion forms the basis to confirm the strength of concrete produced with the characteristic strength adopted in the design of reinforced concrete construction. The European standard EN 206-1, functioning in Poland since 1st January 2004, allows two computational procedures used in the process of monitoring conformity concerning strength. In both methods, the results of the measurements are grouped into series of required number n, and the difference is in the way of creating the series. Series may form overlapping groups or be non-overlapping.

The changes in the Polish standardization concerning the assessment of concrete production and its control of the quality, which were introduced by the standard PN- EN 206-1:2003, have been extensively described by many authors (e.g. Kohutek 2002; Brunarski 2004; Bajorek 2006). There have also been a lot of comments on this standard (e.g. Czarnecki et al., 2004). In literature, one can find the patterns of conformity assessment sheets of specific assortment of concrete (Kohutek 2009; Bajorek, Betlej 2010). However, it is difficult to find the material which analyzes and assesses the risks borne by the manufacturer using the above calculation procedures carried out during the obligatory conformity testing. The results comparison of conformity assessment on compressive strength of concrete, carried out including both ways of results aggregation, were presented for the first time in Poland in 2009 by Zdzisław Kohutek (Kohutek 2009).

Conformity assessment according to PN-EN 206-1:2003

According to PN-EN 206-1:2003 Concrete - Part 1: requirements, properties, production and conformity assessment are carried out within the framework of conformity testing, which, in turn, is one of the components of production control. According to this standard, all standardized concrete parameters whose value is determined using the appropriate equipment or laboratory equipment are subject to conformity assessment. Tests are performed both at the stage of the concrete mixing as well as on the hardened concrete.

Figure1 presents the diagram of conformity testing, designed with the consideration of standard provisions.

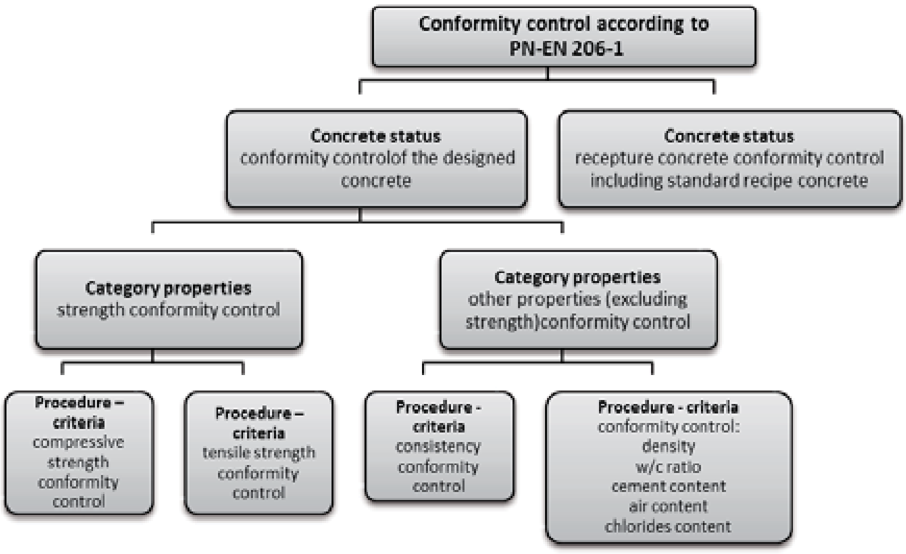

Fig. 1. Conformity control system of concrete properties (Kohutek 2009, s.178).

The article focuses attention on the conformity control of the designed concrete concerning its compressive strength. In the light of the applicable standard, such a control is performed by the manufacturer. The standard describes in detail the plan for the collection, preparation and examination of samples on the bases of which concrete strength parameters test is conducted. Citing provisions of the standard PN -EN 206-1:2003 "any concrete should be subject to the process of production control whose responsibility bears the producer". It is a producer who, having carried out conformity control, ensures a recipient of the quality of concrete available on the market. He declares that the concrete produced in the factory is of appropriate quality determined by the characteristic strength tested on cylindrical or cubic samples.

According to PN EN 206-1:2003 conformity control of concrete compressive strength is carried out with the consideration of two stages of production, initial and continuous. The initial production phase continues until collecting at least 35 results needed to determine a preliminary estimate of the population standard deviation (σ). The standard requires that the manufacturing process, during which results used later to calculate point assessment estimator of parameter σ are collected, lasts longer than three months. After this time, assuming that there has been no more than 12 months, production goes into the stage called continuous production.

Depending on the stage of production and possessing the certificate of production control, appropriate sampling frequency is used. Sampling should be staggered while production. After the first three samples of the first 50 m^3 of production it is not recommended to collect more than one sample from each 25 m^3 of the mixture.

The analysis of conformity criteria of compressive strength

Conformity criteria concerning compressive strength are described in the chapter 8.2.1.3 of the standard PN -EN 206-1:2003. Results of strength measurements are processed according to numerical procedures, defined as criterion 1 and criterion 2 (Table 1). The fulfillment of both criteria simultaneously confirms conformity.

Tab. 1. Conformity criteria concerning compressive strength.

Production	Number „*n*” of test results used to assess conformity	Criterion 1	Criterion 2
		Mean of „*n*” test results (f_{cm}) N/mm^2	Any single test result (f_{ci}) N/mm^2
Initial	3	$\geq f_{ck} + 4$	$\geq f_{ck} - 4$
Continuons	15	$\geq f_{ck} + 1{,}48\sigma$	$\geq f_{ck} - 4$

Source: PN- EN 206-1:2003

The result of test can be the strength value (f_{ci}) obtained from a single cubic or cylindrical sample, or it may be the arithmetic mean of the measurements from at least two samples taken from the same sample of mixture and tested in the same age. In the second procedure of the calculation of (f_{ci}) result, one should skip the values different from the preliminarily calculated mean of more than 15%, unless the analysis of the case reveals any rational reason explaining the omission of a single test result.

Size (f_{ck}) means the characteristic strength defined as the quantile of order 0.05 of the distribution strength general population. This is the value at which concrete class is determined, for example: class C25/30 means that the minimum characteristic strength determined on the cylinder ($f_{ck,cyl}$) size:150x300 mm should be 25 MPa, and on the cube ($f_{ck,cube}$) of the edge 150 mm – 30 MPa.

The criterion 1 uses the arithmetic mean (f_{cm}) calculated from the series of results of three numbers at the initial production stage or 15 numbers at the continuous production. Series are created from the consecutive results (f_{ci}) and may be separate (Fig.2) or may overlap (Fig.3).

f_{c1}	f_{c2}	f_{c3}		f_{c4}	f_{c5}	f_{c6}		f_{c7}	f_{c8}	f_{c9}

Fig. 2. Grouping scheme in the non- overlapping results sequence.

f_{c1}	f_{c2}	f_{c3}		f_{c2}	f_{c3}	f_{c4}		f_{c3}	f_{c4}	f_{c5}
1st series				2nd series				3rd series		

Fig. 3. Grouping scheme in the overlapping results sequence.

The arithmetic mean, calculated on the basis of overlapping sets of results (Fig.3) is a moving average sleeking a series of research results obtained during the production process.

The note, namely, the use of criteria to the overlapping results increases the risk of their rejection, is included in the standard. This would mean a higher risk attributable to the manufacturer. However, the calculations based on the overlapping results allow earlier detection of non-conformity. As a result, a producer has the chance to take earlier corrective action. It should be noted that the results are obtained with a delay compared in relation to concrete mixture production. Samples of hardened concrete are subjected to compression on 28th day of curing, and if the strength is specified for different age groups, then samples test is carried out at the specified age. This means that the producer receives a signal of the lack of conformity after about one month from the time when the concrete came to the site. If the assessment of conformity is based on non-overlapping sets of results, then this time is even longer. It is necessary to collect the required number of results in the series, i.e. 3 or 15. This may take several more days or even another month, depending on the volume of the produced concrete mix. The delay in the detection of non-conformity is, in turn, detrimental to the recipient. It increases the risk of purchasing concrete of lower class than that declared by the manufacturer. This may cause disadvantageous effects.

The standard PN-EN 206-1:2003 contains also the information that the conformity criteria are based on the overlapping results. It can be assumed that the producer, who has laconic information about the origin of developing the criteria and a warning about the increased risk of rejection of an "good" concrete batch, shall decide on the procedure for assessment of conformity based on overlapping sequences of test results. The priority may turn out to be economic reasons rather than safety.

The criterion 2, identical at both stages of production, is reduced to checking the minimum result in the series and it does not matter whether series overlap or not. Moreover, the standard also imposes the obligation to check the results dispersion of measurements of the compressive strength. When production goes into a continuous phase, it is necessary to control the spread of results using the standard deviation of s_{15} of the last 15 test results. The criterion 1 used at this stage of production uses the standard deviation σ, calculated from the results collected during the initial production (at least 35 results collected over a period longer than three months, obtained in the directly preceding period of production, during which conformity is to be checked). The control of the dispersion of results is based on checking whether the estimator s_{15} “appropriately” estimates the adopted standard population deviation (σ) confidence level $1 - \alpha = 0.95$. Knowing the value of σ one can easily convert a relation (1):

$$\sqrt{\frac{(n-1)s_n^2}{\chi^2_{1-\frac{\alpha}{2},n-1}}} \leq \sigma \leq \sqrt{\frac{(n-1)s_n^2}{\chi^2_{\frac{\alpha}{2},n-1}}} , \qquad (1)$$

describing the confidence interval for the standard deviation, where after having calculated the quantile of order $1 – \alpha/2$ from the chi–squared distribution and the number of degrees of_freedom $df = n – 1$, for $n = 15$ and $\alpha = 0.05$, one obtains the relation:

$$0{,}63\sigma \leq s_{15} \leq 1{,}37\sigma \qquad (2)$$

If the condition (2) is not fulfilled, one must specify a new estimate σ from the last available 35 test results. A new estimate σ determined in this way is to be used for the next period of conformity assessment.

Producer’s risk assessment while using overlapping and non-overlapping sequence of results

According to the comments on EN 206-1, published in various sources (Harrison et al. 2001 Caspeele 2010) the assessment of concrete compressive strength procedures according to conformity criteria has been developed on the basis of data processing obtained by computer- generated random values (simulation method) and the analysis of the actual production of a few specific plants in Europe. Mathematical development of conformity criteria was described in detail by Lesław Brunarski (Brunarski 2009).

In the statistical procedures of quality control, along with accepted sampling inspection plans, Operating Characteristic Curve (OC-curve) is used. These OC functions represent the dependence of the probability of acceptance of tested product batches on the actual quality level of the batch, measured by defectiveness. One can assess the risk borne by the concrete producer by means of the Operating Characteristic if the actual level of production quality is known.

Based on the OC curves current values of the coefficients in the conformity criteria have been selected (Taerwe 1988).

Oparating Characteristic has been used by the author to investigate the problem, namely, how the aggregation of results affects the risk of rejection of a "good" batch of concrete.

Due to the complexity of the analytical structures, the construction of OC curves was performed using the Monte Carlo simulation.

Contemporary building standards treat the strength of the material as a random variable of normal distribution. Hence, this distribution has been adopted as the starting theoretical distribution.

First of all, two sets of series of random numbers conforming to the standard normal distribution were generated. For this purpose a random number generator implemented in the Statistica program was used. The number of three-element series amounted to 100 000 whereas the number of 15-element series was 69905. This data made the base for the non-overlapping sequence of results. The number of overlapping groups created on this basis amounted to 299 998 and 1 048 561.

Taking concrete class C25/30 as a model and determined standard deviation (*σ = 2, 3, 4, 5 and 6 MPa*) five different normal distributions were obtained. The probability of acceptance for the compound conformity criterion according to PN-EN 206-1 was calculated with a fixed defective fraction *w*, including the initial and continuous stage of production.

The shape of curves obtained for the initial production OC are shown in figure 4 and figure 5.

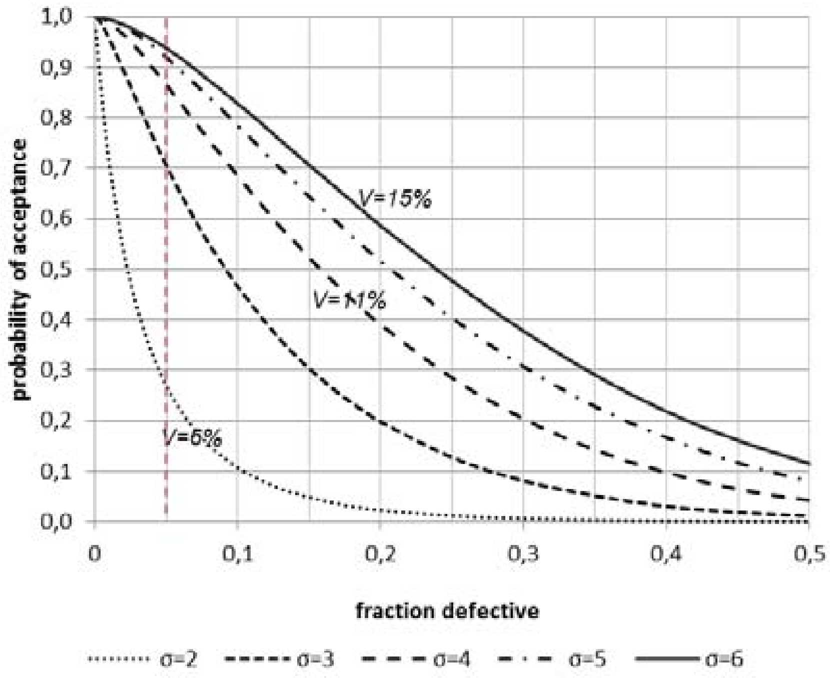

Fig. 4. OC curves compound conformity criterion in accordance with PN-EN 206-1 and different dispersions of normal distribution for the non-overlapping groups of results (number of results in a series n = 3).

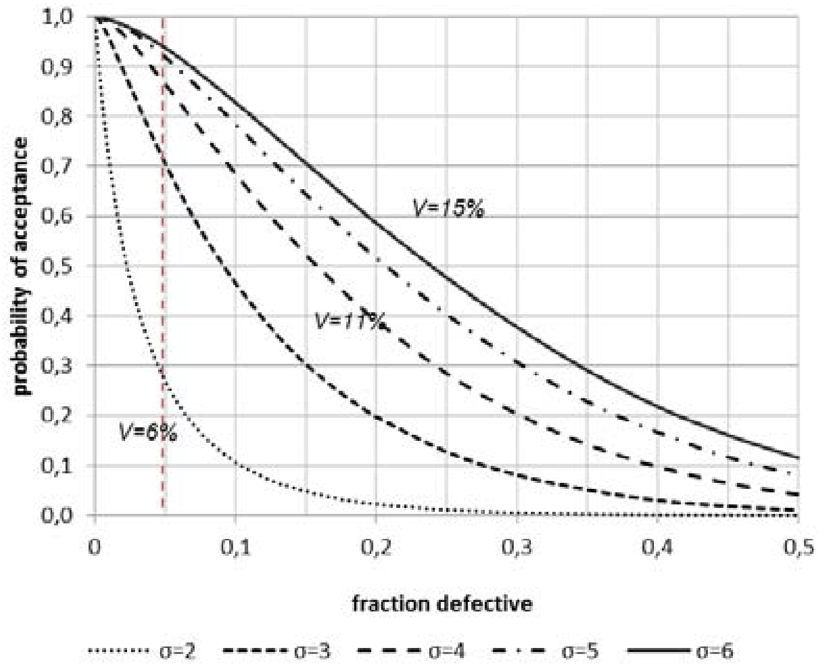

Fig. 5. OC curves compound conformity criterion in accordance with PN-EN 206-1 and different dispersions of normal distribution for the overlapping groups of results (number of results in a series n = 3)

Comparing the graphs presented above, it can be concluded that the way of result aggregation did not affect the shape of the OC curve. The exact values of obtained probabilities of acceptance at a fixed defectiveness level of, for example, 5%, can be traced by analyzing the data presented in Table 2. The differences between the values are minimal, of the order of one or a few ten-thousandths.

Tab. 2. Probability of acceptance of a double conformity criterion in accordance with PN-EN 206-1:2003 at a constant batch defectiveness *w* = 0.05 at the initial production phase (n = 3).

The way of grouping	Standard deviation (V – coefficient of variation)				
	σ = 2 V=6%	σ = 3 V=8,6%	σ = 4 V=11%	σ = 5 V=13%	σ = 6 V=15%
Non-overlapping groups	0,2695	0,7057	0,8650	0,9192	0,9375
Overlapping groups	0,2690	0,7053	0,8658	0,9192	0,9376

Source: own_calculations

Owing to the lack of significant differences, one of the distributions has been selected, on the example of which the observed phenomena were presented. Normally the level of concrete compressive strength measurements diversity is about 11 - 13%, so for such a coefficient of variation a sample comparison of OC curves was presented, confirming the lack of influence of the results grouping on the probability of acceptance at the initial production phase (Fig. 6). The situation is the same for other analyzed distributions.

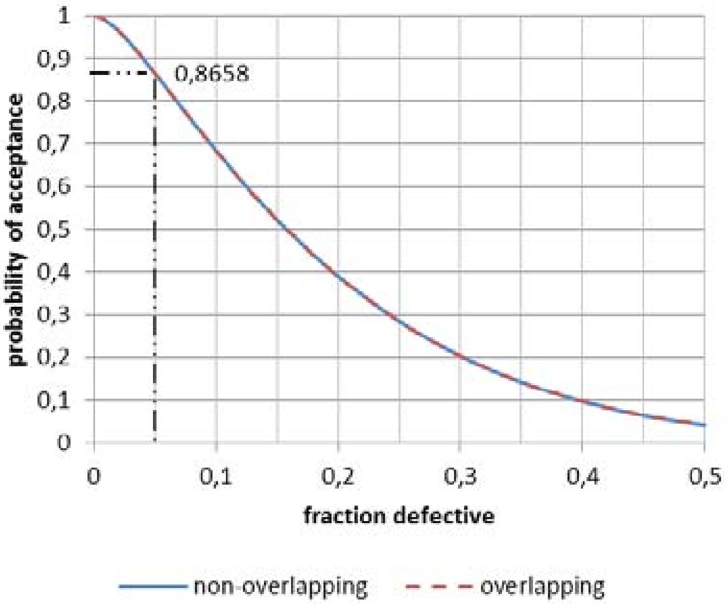

Fig. 6. OC curves for the overlapping and non-overlapping groups of results (n = 3, σ = 4).

Whereas, the dispersion of results clearly modifies the shape of graphs. Increasing dispersion leads to reducing the risk of rejection of a "good" batch of concrete. This problem was noticed by Woliński and Skrzypczak (Woliński, Skrzypczak 2006). In their view, such a situation may discourage producers to take steps aiming to ensure uniformity of production at the expense of increasing the average strength and it increases the risk of a recipient connected with the purchase of concrete batch of underestimated quality.

While using the conformity criteria at a continuous production stage it has not been observed that the aggregation of the results would increase the producer risk. The probability of acceptance calculated using a compound criterion: $f_{cm} \geq f_{ck} + 1{,}48\sigma$ and $f_{c,min} \geq f_{ck} - 4$ with respect to non-overlapping and overlapping groups was almost identical. OC curves are shown in Figure 7 and 8.

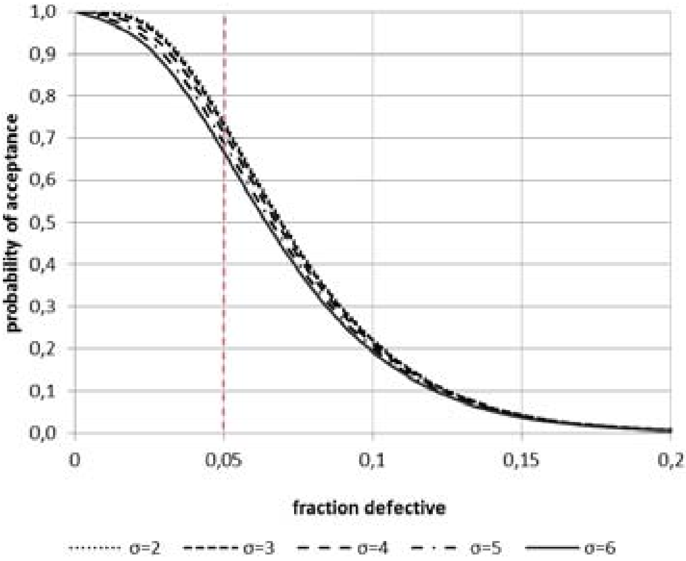

Fig. 7. OC curves of a compound conformity criterion in accordance with PN-EN 206-1 and various dispersions of normal distribution for the non-overlapping groups of results (number of results in a series n = 15)

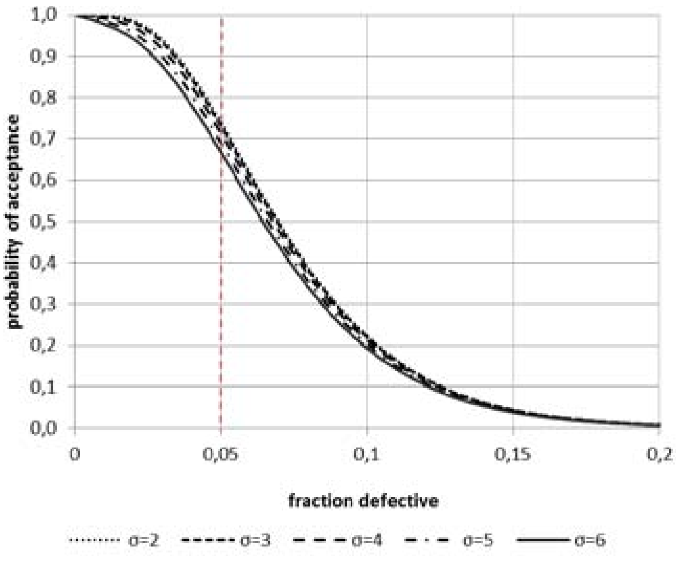

Fig. 8. OC curves of a compound conformity criterion in accordance with PN-EN 206-1 and various dispersions of normal distribution for the overlapping groups of results (number of results in a series n = 15).

The exact values of probability of acceptance with defective fraction of 0.05 are set in Table 3.

Tab. 3. Probability of acceptance of a compound conformity criterion in accordance with PN-EN 206-1:2003 at a constant batch defectiveness $w = 0.05$ at the continuous production phase (n = 15).

The way of grouping	Standard deviation (V – coefficient of variation)				
	σ = 2 V=6%	σ = 3 V=8,6%	σ = 4 V=11%	σ = 5 V=13%	σ = 6 V=15%
Non-overlapping groups	0,7373	0,7286	0,7111	0,6886	0,6658
Overlapping groups	0,7391	0,7305	0,7127	0,6899	0,6672

Source: own calculation

The differences in the values of probability of acceptance are already slightly higher than in the case of the criteria used for the initial production stage, but still minimal, of the order of several thousandths. Should one consider this difference as vital, more beneficial calculation procedure would be the one that is based on the overlapping sets of results.

OC curves of compound conformity criteria applied to overlapping and non-overlapping groups of results, presented in one coordinate system run almost identically to the same standard deviations, the differences are hardly noticeable. A graphical presentation of all these graphs, almost skipped in this article, has been limited to one. An example of the OC function graph of a compound conformity criteria for the standard deviation of 4 and $n = 15$, with a more detailed scale is shown in Figure 9.

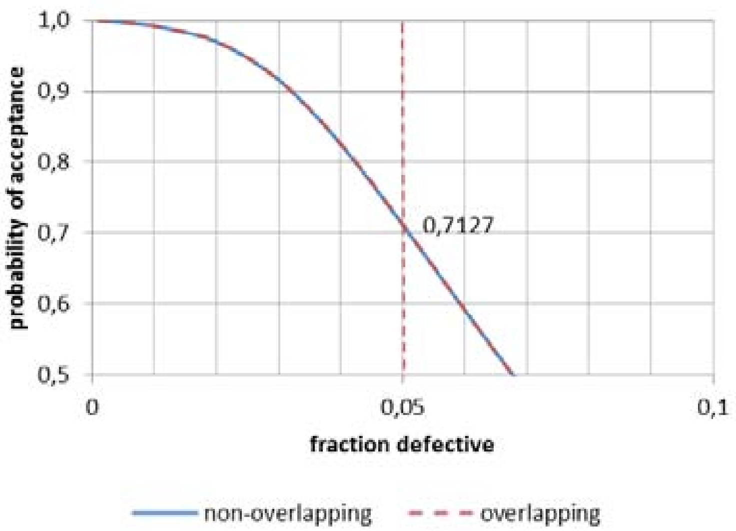

Fig. 9. OC curves for the overlapping and non-overlapping groups of results (n = 15, σ = 4).

In addition, it should be noted that the author has received the values of probability of acceptance very close to the results presented by Woliński S. Skrzypczak I. (Woliński, Skrzypczak 2006). However, the information on how to create a series of subsequent results was not provided in the description of the algorithm for calculating operating characteristic.

The calculations and analysis of the results let us draw the conclusion that the aggregation of overlapping or non-overlapping results in a series does not affect the producer's risk connected with the rejection of a "good" batch of concrete.

A different conclusion is drawn by Zdzislaw Kohutek (Kohutek 2009). He estimates as "more sensitive " sequence of overlapping results. As far as disjoint setting of series of subsequent results is concerned, he writes that "..it provides the added value since a possible single result, deviating excessively, spoils averaging only once, not repeatedly".

In the case of analyzing the data, including the empirical one, one can talk about the method sensitivity while comparing the frequency of detected relative nonconformities, not as described in the cited report, absolute frequencies. In the case of creating a sequence of overlapping series, the number of results increases according to the formula:

$$N^* = n \cdot (N - 1) + 1 \tag{3}$$

where:

n – the size of series ,
N – number of non-overlapping series,
*N** – number of overlapping series.

The increased number of series *N** must be included in the calculation of the frequency of detected relative nonconformities.

Conclusions

1. The standard PN-EN 206-1:2003 allows conformity assessment of concrete compressive strength according to two calculation procedures. Consecutive measurements are grouped in the series of the required number of *n*. These series may overlap or be separate (as non-overlapping groups). The choice of a procedure depends on the producer.
2. Probability of acceptance of a compound conformity criterion in accordance with PN-EN 206-1:2003, determined using the Monte Carlo method, does not depend on the grouping of results in the series.
3. The results of calculations performed on randomly generated "large" data sets provide the evidence that conformity control of concrete compressive strength carried out on overlapping groups of results does not increase a producer’s risk.
4. The author recommends to check conformity control of concrete compressive strength on the basis of overlapping sequences of results due to the possibility of early detection of non-conformity, which makes an opportunity to take corrective action as soon as possible.

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