MODELING OF CRACKING BEHAVIOUR OF R/C-MEMBERS BASED ON THE BOND-SLIP RELATION FOR BONDED REINFORCEMENT

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Summary:
In this article is presented the new approach for design procedure for crack width calculation, received on the basis of experimentally-theoretical research of axially-loaded reinforced concrete tensile elements. This new approach (calculation method) based on studying of the mechanism of interaction and internal forces redistribution between reinforcement and surrounding concrete. It was possible to establish laws of crack formation in reinforced concrete elements, to receive the equations to establish strain distributions of reinforcement and tension concrete on the length of an element and to receive an analytical equation for definition of the length of the strain redistribution zone for the various bonding conditions, to receive a relation for calculation of average crack width (w_m).

Keywords: crack width, reinforcement, concrete, strain distributions, length of the strain redistribution zone

Introduction

Currently, there is a large number of calculation methods proposed for determining of crack resistance of reinforced concrete elements. Based on analysis performed by the authors the following calculations methods were proposed:

- Empirical relationships (ACI 224.2R-86 1986; Gergely i in., 1968, Mulin 1974, Gusha 1976, etc.);
- Expressions based on the theory of fracture mechanics (Piradov i in., 1991, Guzeev 1991, Oh i in.1987, Shah 1995, etc.);
- Expressions based on the theory of «tension stiffening» (TSE) (CEB-fib MC 2010, EN 1992-1, Pedziwiatr 2008, SNB 5.03.01-02, Murashev 1962, Nemirovskij 1970, etc.);
- Relationships obtained from the analysis of the stress-strain state of reinforcement and surrounding concrete along the length between sections with cracks (Holmberg 1984, Farra 1992, Noakowski 2004, Alvares 2004, a proposed approach, etc.).

In the modern theory of crack resistance of the concrete element last two approaches are most widely used. Expressions of these groups what described the mechanism of crack formation are similar. The main difference between of two calculation methods consists of to determining the strains distribution and crack space. In the expressions derived on the basis of TSE-method, the strain difference of reinforcement and concrete between sections with cracks is determined by multiplying the value of strain in the cracked cross section and dimensionless coefficient of ψ, which characterizes the uniform distribution of the mean strains along the length between sections with cracks. The mean crack spacing (s_{rm}) traditionally obtained as follow. In general, the expression for determining s_{rm} may be obtained from the equilibrium conditions, compiled for the block concluded between the cross-section with a crack and section located in the middle of the block, and is corrected multiplying by empirical coefficients.

The expressions obtained in the framework of the fourth group models can most fully describe the real development of cracks, because they are based on the physical laws of interaction of materials under load. Obviously, the accuracy of the results obtained using the expressions of this group, directly depends on the adequacy of the idealized diagrams used in calculation models to describe physical laws (strain-stress diagram, bond-slip diagram).

Basic provisions of the proposed approach

General view on crack development in RC-elements

The behavior of reinforced concrete elements under tensile load will be considered on the example of a symmetrically reinforced concrete elements subjected to axial tension. Tensile load applied on the section in the end of element, which conditionally can be considered as crack. The whole tensile load is perceived by reinforcement bar. For intermediate points what situated between end section and middle section of an element tensile force is transferred from the reinforcement to the concrete by bond forces. Such mechanism of strain redistribution between the reinforcement bars and the surrounding concrete corresponds to a gradual reduction of the value of strains of reinforcement and, consequently, to an increase in strains of concrete diagrams (Fig. 2.1a).

In a section, located at a distance of l_t from the end of element reinforcement and concrete strains are equal ($\varepsilon_s=\varepsilon_{ct}$). Thus, along the length of the reinforced concrete element under the axially applied tensile load two characteristic zones can be identified (Fig. 2.1):

1. zone of equal strains – $\varepsilon_s=\varepsilon_{ct}$;
2. zone of strain redistribution – l_t.

In accordance with the mechanism of strain-stress redistribution presented above, the formation of cracks in this element can occur only within the zone of equal strains, when tensile strains in concrete reaches ultimate value (ε_{ct1}) (see Fig. 2.1).

The formation of the first(-s) crack in any of sections of zone 1 is equally probable, because throughout its length is a homogeneous stress-strain state take place. The most unfavorable case is observed when the smaller number of cracks is formed along the length of the element. In this case, the crack width value will be the maximum. Suppose that will be formed only one crack in the cross section, located exactly in the middle of equal strain zone (zone 1, Fig. 2.1).

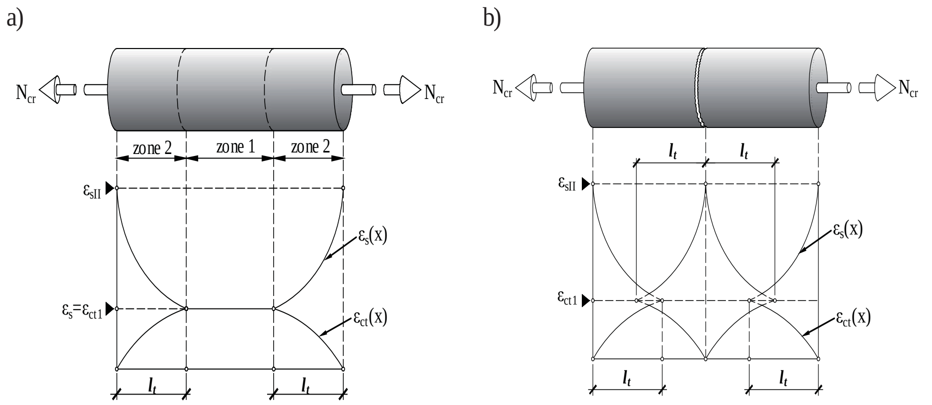

Fig.2.1. Strain distribution along of concrete element with allocation of characteristic zones:
a) –before cracking; b) – after cracking;

The development of cracks is progressive and the cracks number increases as the load increases until a stabilized cracking condition is reached. The stabilized cracking is characterized by the overlapping of strain redistribution zone of two neighboring cracks (Fig. 2.1b). In any sections along the length of block observed a difference in the strains, which leads to slippage of reinforcement. It should be noted that the formation of new cracks in the already allocated blocks is possible only when strains in the middle section along the length of the block reached the ultimate value (ε_{ct1}).

After a stabilized cracking condition is reached further increase of tensile load leads to an increase in the difference between values of the strains of reinforcement and concrete in each section along the length of the element, and as a consequence, increase reinforcement slippage, which in turn causes an increase in crack widths (see Fig. 2.2).

The length of strain redistribution zone

As follows from the mechanism of stress-strain state formation in reinforced concrete elements at different loading stages the key factors what influencing at development of cracks are as follows:

1. length of the strain redistribution zone l_t;
2. functions, what describing the distribution of strains of reinforcement $\varepsilon_s(x)$ and the concrete $\varepsilon_{ct}(x)$ along the length of the redistribution zone.

Under the length of strain redistribution zone authors mean a conditional length, which for each stage of loading is required to transfer a part of the tensile force from the reinforcement to the concrete by bond and measured between section with a crack, (where $\varepsilon_{sII}=N/(A_sE_s)$ and $\varepsilon_{ct}=0$) and the section with equal strains of reinforcement and concrete ($\varepsilon_s=\varepsilon_{ct}$). Strain redistribution zone length depends on the following parameters:

– value of axially applied load;
– bond conditions;
– the geometric parameters of RC-element;
– mechanical characteristics of concrete and reinforcement.

Introduction of "conditional length" concept is based on and justified by the fact that the length of this zone increases as applied load is increased and in some cases may exceed the length of the reinforced concrete element under consideration. (see Fig. 2.2).

Basic assumptions of the proposed approach

The design crack width can be determined using the following expression:

$$w = \int_L \left[\varepsilon_s(x) - \varepsilon_{ct}(x) \right] dx, \tag{2.1}$$

where L – block length between cracks; $\varepsilon_s(x)$ – function, describing the reinforcement strain distribution; $\varepsilon_{ct}(x)$ – function, describing strains distribution of surrounding concrete in tension;

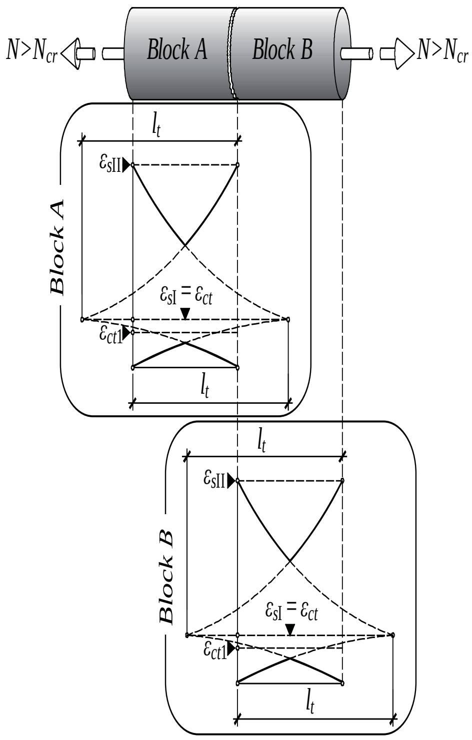

Fig.2.2. Strain distribution in reinforcement and concrete along the length of an element (calculation case: redistribution zone length is longer than the block allocated by cracks)

In addition to the functions $\varepsilon_s(x)$ and $\varepsilon_{ct}(x)$ is necessary to determine the final (maximum) length of the block between neighboring cracks, where crack development is impossible. According to the provisions given in (2.1), the formation of following crack in the block is possible if at least in one section along the length the value of reinforcement and concrete strains are equal, i.e. $\varepsilon_s=\varepsilon_{ct}$. Such a case is possible if the block length L is equal to the twice length of the strain redistribution zone when tensile load is equal N_{cr}, i.e.($L=2l_{t,cr}$). After the crack formation the block will be divide into two new blocks with the length $l_{t,cr}$ and expression (2.1) takes the following form:

$$w = \int_{l_{t,cr}} \left[\varepsilon_s(x) - \varepsilon_{ct}(x)\right] dx \tag{2.2}$$

The numerical algorithm was proposed for evaluation of analytical expressions for calculating the length of the strain redistribution zone l_t and functions $\varepsilon_s(x)$ and $\varepsilon_{ct}(x)$ a planar coordinate system with origin in the cross-section with condition $\varepsilon_s=\varepsilon_{ct}$ is accepted. The positive direction of the horizontal axis taken in the direction of the cross section with a crack. Using an iterative numerical method for the elementary interval Δx, allocated by along the horizontal axis is searched for distribution of concrete and reinforcement strains, satisfying the of equilibrium equations (see Fig. 2.3):

$$\begin{cases} \sigma_{s\,i} - \sigma_{s\,i-1} - \Delta x \cdot \left(\dfrac{\tau_{b\,i} + \tau_{b\,i-1}}{2} \right) \cdot \dfrac{4}{\varnothing_s} = 0 \\ \sigma_{ct\,i-1} - \sigma_{ct\,i} - \Delta x \cdot \left(\dfrac{\tau_{b\,i} + \tau_{b\,i-1}}{2} \right) \cdot \dfrac{4 \cdot A_s}{\varnothing_s} \cdot \dfrac{1}{A_{ct,netto}} = 0 \end{cases} \tag{2.3}$$

Solution of the equations (2.3) can be determined with using following laws:

– for the reinforcement – by the elastic part of relationship “σ_s-ε_s”;
– for the concrete – by the ascending branch of the strain diagram of concrete in tension;
– for the bond– according to the diagram CEB-fib-MC 2010.

a) b)

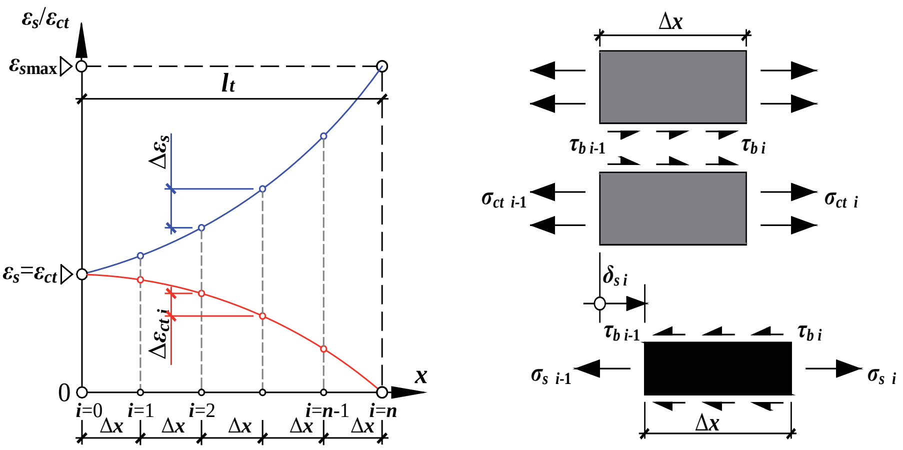

Fig. 2.3. To making the calculating algorithm:
a) – to definition of increments of stains;
b) – conditions of equilibrium for an elementary segment Δx.

The condition for complete the calculated procedure of the algorithm to determine the position of the cross section for which would have the next equations:

$$\varepsilon_{ct} = 0 \quad \text{and} \quad \varepsilon_s = \frac{N}{A_s \cdot E_s} \tag{2.4}$$

Numerical investigation using of special computer program containing the calculating procedures of presented algorithm was performed. On the basis of a computer program containing the calculating procedures of the presented algorithm was performed computational experiment, which program included determination of analytical expressions to determine the length of there distribution zone, as well as functions describing the strains redistribution of reinforcement $\varepsilon_s(x)$ and the concrete in tension $\varepsilon_{ct}(x)$ within this zone. During the computational experiment was carried out varying of the following parameters:

The following parameter was carried out varying of following parameters:

– diameter of reinforcement – 10 ... 40 mm;
– type of surface of bars – ribber bars, smooth bars;
– effective reinforcement ratio – 0.25...4.0%;
– the average strength of concrete – 1.3...2.9 MPa.

Based on the analysis results of computer simulation for the calculation the length of the strain redistribution zone following expression was obtained:

$$l_t = k_p \frac{N_{ult}}{u \cdot \left(1 + \rho_{eff} \cdot \alpha_E\right)} \cdot \sqrt{\frac{N}{N_{ult}}} \tag{2.5}$$

where

k_p –factor, which takes in account bond properties of reinforcement, mm^2/N;
N_{ult} – value of ultimate tensile force, kN;
u – perimeter of contact between reinforcement and surrounding concrete;
ρ_{eff} – the effective reinforcement ratio;
A_s – the reinforcement area;
$A_{ct,eff}$ – the effective tension area of concrete;
α_E – a ratio of modules of elasticity of reinforcement and concrete.

To describe the strain distribution of reinforcement and surrounding concrete the following equations were received:

$$\varepsilon_s(x) = \varepsilon_{sII} \cdot \left[a \cdot \left(\frac{x}{l_t} \right)^{\frac{1+\alpha}{1-\alpha}} + b \right] \tag{2.6}$$

$$\varepsilon_{ct}(x) = \varepsilon_{sII} \cdot \left[1 - \left[a \cdot \left(\frac{x}{l_t} \right)^{\frac{1+\alpha}{1-\alpha}} + b \right] \right] \cdot \rho_{eff} \cdot \alpha_E \tag{2.7}$$

where e_{sII} – strain of reinforcement assuming a cracked section; a and b – dimensionless coefficients are defined by:

$$a = \frac{1}{1 + \rho_{eff} \cdot \alpha_E} \quad \text{and} \quad b = \frac{1}{1 + \frac{1}{\rho_{eff} \cdot \alpha_E}} \tag{2.8}$$

Inserting obtained relationships into (2.2) and performing the appropriate transformations, expression for determining crack width takes the form:

$$w_m = 0{,}15 \cdot k_p \cdot \varepsilon_{sII} \cdot \frac{\varnothing_s}{1 + \rho_{eff} \cdot \alpha_E} \cdot \sqrt{\frac{\sigma_{sII}}{f_{yk}}} \cdot \left[1 - \left(1 - \frac{1}{2} \cdot \sqrt{\frac{f_{ctm}}{\sigma_{sII} \cdot \rho_{eff}}} \right) \right] \tag{2.9}$$

where

σ_{sII} – reinforcement stress for cracked section; f_{yk} – characteristic strength of reinforcing steel, MPa; f_{ctm} – average tensile strength of concrete, MPa.

Experimental verification of proposed analytical model

For verification of proposed analytical model several tests series of axially reinforced concrete elements have been executed. At manufacturing of test specimens for various experimental series it was made by variation in reinforcing parameters (diameter of reinforcement (Øs=20, 25 and 36mm)); type of a surface of reinforcement (smooth bars and ribber bars) and strength characteristics of steel reinforcement and concrete of specimens.

To measurement of reinforcement bar strain within embedding length of concrete it was applied strain-gage method. As primary measuring devices have been used strain gages with base 5,0mm. Strain-gages were pasted in the grooves executed on a lateral surface of reinforcement bars along longitudinal edges of a profile, in chessboard order with step 50mm that has allowed equipping each bar with 41 gages.

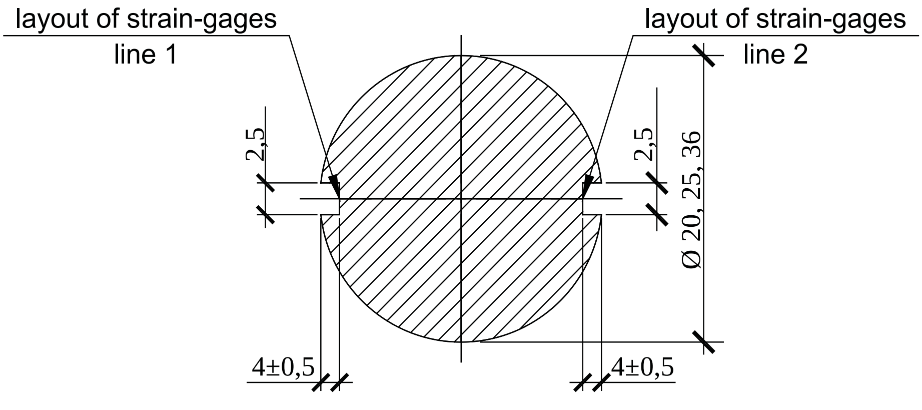

Fig. 3.1. Layout of strain-gages on the reinforcement bar

During the tests of experimental specimens at each stage registration of following parameters was made:

– reinforcement bar strains on embedded length in the concrete on the basis of strain-gage method at use of a measuring computer complex;
– crack width at use of microscope.

The results of comparison of experimental and analytical strain distributions of reinforcement for stages accordingly before and after crack formation are presented in Figure 3.2. It is necessary to notice that at the description of strain distribution for reinforcement $\varepsilon_s(x)$ and concrete $\varepsilon_{ct}(x)$ tensile load and of position cracked section were accepted according to experimental data.

The comparative analysis of the experimental and analytical data (Fig. 3.2) gives the grounds to draw a conclusion about legitimacy of hypotheses and the preconditions accepted by working out of model, and also about adequacy of dependences $\varepsilon_s(x)$ and $\varepsilon_{ct}(x)$, proposed for the description of stress-strain condition of reinforcement and the tension concrete of axially reinforced concrete element. Comparison of values of average crack width received during carrying out of tests, to the corresponding values calculated according to various calculating procedures: to a proposed method and standard method EN-1992-1 (2004), it is executed in Figure 3.3. The comparative analysis of results of calculation shows that the Eq. (2.9) proposed for calculation of the average value of crack width, provides comprehensible reliability of calculating values in relation to the experimental data. Besides, character of change of values of average crack width calculated according to proposed relation, is most close to the character taking place in the experiment.

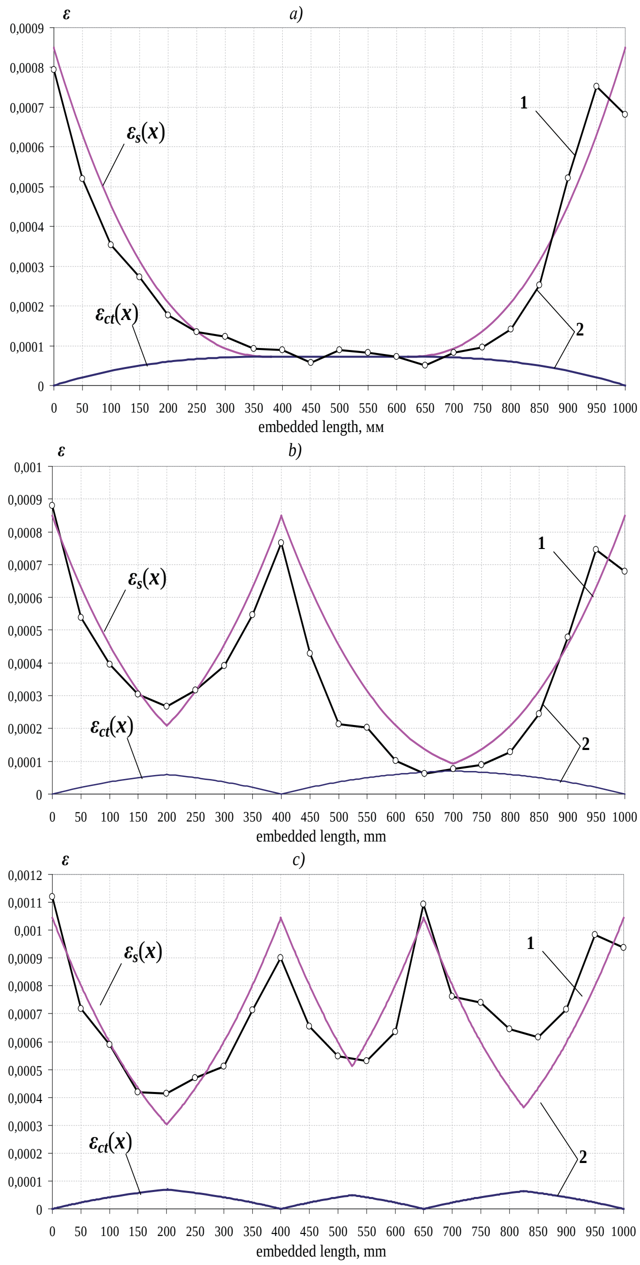

Fig. 3.2. Comparison of strain distributions, received experimentally (1) and analytically (2), for the specimen with following characteristics – 1Ø25S400; ρ_{eff}= 0,015; f_{ctm}=2,47N/mm^2
a) – before crack formation of (N=80kN); b) – after first crack formation (N=80kN);
c) – after second crack formation (98,4kN)

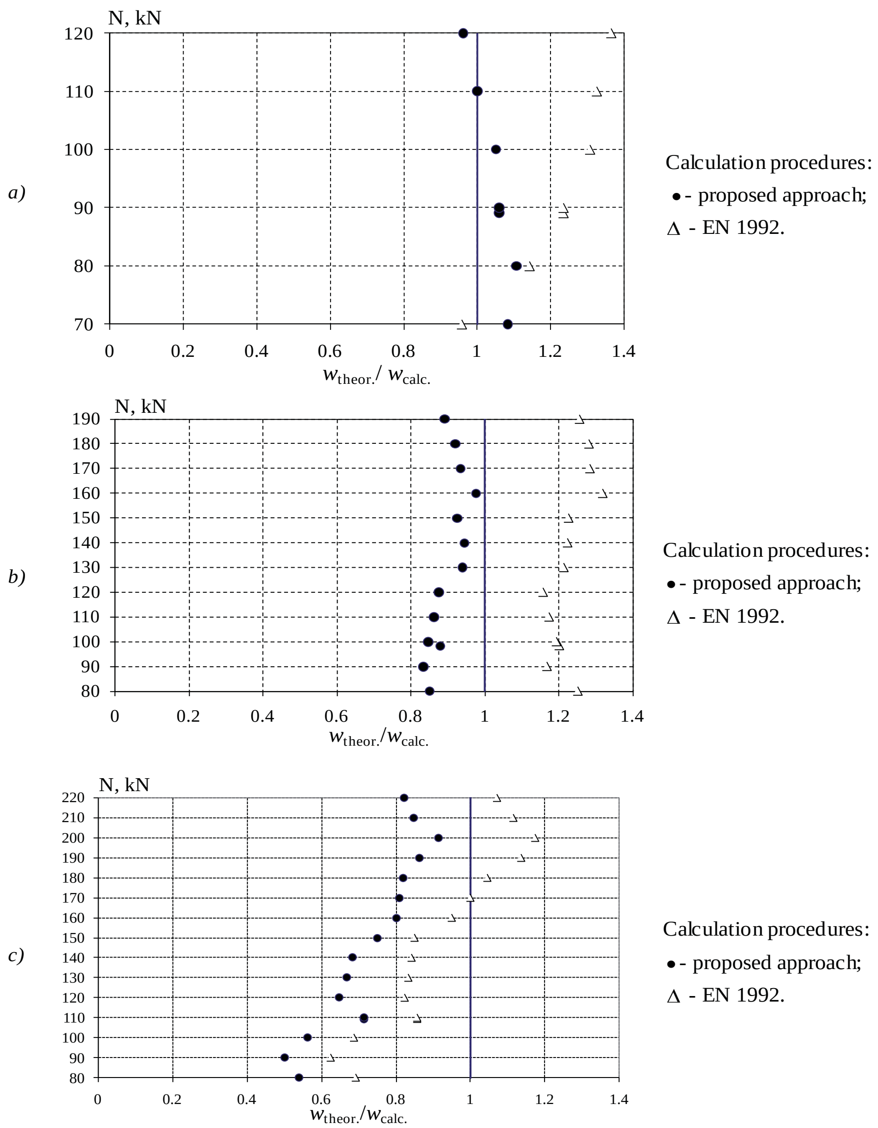

Fig.3.3 Comparison of average values of crack width
a) - experimental specimen with following characteristics – 1Ø20S400; ρ_{eff}= 0,01; f_{ctm}=2,47N/mm^2;
b) - experimental specimen with following characteristics – 1Ø25S400; ρ_{eff}= 0,015; f_{ctm}=2,47N/mm^2;
c) - experimental specimen with following characteristics – 1Ø36S400; ρ_{eff}= 0,03; f_{ctm}=2,47N/mm^2;

Conclusion

Proposed concept based on an analysis of strain-stress state of concrete and reinforcement qualitatively characterizing the conditions of crack formations in RC-elements under axially applied tensile load.

Based on the general provisions of the concept were obtained:

– equations describing the strain distribution of reinforcement and concrete along the length of the strain redistributions zone at any loading stage;
– relationship of the crack width and strains distribution in the reinforcement, the geometric parameters of tensile RC-element cross-section.

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