

ANALYSIS OF SHEAR CAPACITY OF STEEL FIBER REINFORCED CONCRETE BEAMS

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Summary:

Fibers may be used in the flexural members like beams or slabs as a full or partial replacement of flexural or shear reinforcement. In the last case we have to do with combined reinforcement for shear. The methods for determining the shear capacity for SFRC elements are defined in scientific literature and also in Model Code 2010 and Russian Standard. In the paper there are presented and discussed two recent standard models and methods of calculation of shear loads in steel fiber reinforced concrete beams with combined flexural and shear reinforcement. Some chosen results of experimental investigation on shear capacity in SFRC beams including own research of flexural SFRC members tested under flexure are also presented and discussed. The tests results confirmed that steel fibers in reinforced concrete beams can partially replace (in certain cases) the traditional steel stirrups calculated for shear.

Keywords: Steel fiber reinforced concrete, shear load, shear capacity, Model Code 2010

Introduction

Near to support of the beams shear forces have significant value, and at the same regions can occur the bending moments. As a result, of shear forces exist shear stress and the bending moment causes the normal stress (compression and tension), the summation of the stresses working are formed principal stresses tension $s_1 = s_{\max}$ and compression $s_2 = s_{\min}$. In the reinforced concrete element compressive principal stresses are generally safely transferred through the concrete, which is characterized by high compressive strength. Principal tensile stresses tend to reach values exceeding the tensile strength of concrete. Through the use of transverse reinforcement (stirrups or bent bars), the most shear forces are safely transferred.

In the conventional RC beams steel stirrups work effectively only after formation of diagonal cracks. Their main role is to take the tensile stresses from shearing. Transfer of shear stress in the gaping crack is through the interaction of the four components of the shear force transfer: tensile component of longitudinal reinforcement, the resultant of compressive stress in the concrete, the interlocking aggregate effect and the resultant tensile stress in the stirrups.

The use of conventional steel stirrups causes additional costs associated with the consumption of reinforcing steel, installation costs for construction, during the formation of beam short distance between the stirrups also cause technological problems of uniform distribution of the aggregate in the concrete mix, which may lead to presence of voids and weak bond between the concrete and the reinforcing rods.

An alternative solution might be to apply in concrete beams distributed fibers. In practice the use of such a solution is still not sufficiently confirmed by research. Recently such studies have been undertaken on a large scale for single span beams (Aoude et al. 2012), (Haslam 2011), (Dinh et al. 2010), (Dinh 2009), (Salò, Marciukaitis 2007), (Parra-Montesinos 2006), (Domanski, Czkwianianc 2004); (Hanzlová et al. 2011). At the Department of Building Structures of the Bialystok University of Technology a research was conducted on the effects of steel and basalt fibers for single-supported beams (Krassowska, Lapko 2011) and also for two-span beams (Krassowska, Lapko 2012, 2013).

Theoretical evaluation of shear load in steel fiber reinforced concrete beams

The shear resistance of conventionally RC flexural elements are calculated using model of truss analogy according to the standard PN-EN 1992. The appropriate calculation formulas determining the capacity of:

$V_{Rd,c}$ - Design shear resistance of the member without shear reinforcement (effect of design strength of concrete for tension and some other effects);

$V_{Rd,s}$ - Design value of shear force which can be sustained by the yielding shear reinforcement;

$V_{Rd,max}$ - Design value of maximum shear force which can be sustained by the member, limiting by crushing of the compression concrete struts (in the truss model).

Unfortunately Eurocode does not provide how to calculate the elements made of SFRC (with steel fibers). Methods for determining the shear capacity of SFRC elements are defined in (Model Code 2010), (Russian Standard 2007) and some papers (Lim and Oh 1999) and also by Aoude et al 2012).

Shear capacity of SFRC members according to Model Code 2010

With regard to the behavior in tension, which is the most important aspect of SFRC, various test methods are possible.

Bending tests can be carried out aiming at determining the load-deflection relation. The results can be used for deriving the stress – crack width relations by inverse analysis, performing equilibrium calculations for numerous crack openings. Nominal values of the material properties can be determined by performing a 3-point bending test on a notched beam as showed in Fig. 1. The deformation is generally expressed in terms of Crack Mouth Opening Displacement (CMOD).

The characteristic value of the residual ultimate tensile strength of SFRC is calculated according to the formula:

$$f_{Ftuk} = f_{Fts} - (w_u / CMOD_3) (f_{Fts} - 0,5 f_{R3} + 0,2 f_{R1}) \geq 0 \quad (1)$$

where: w_u is the maximum crack opening accepted in structural design, f_{Fts} serviceability residual strength (post-cracking strength for serviceability crack opening), f_{R3} residual strength of fiber reinforced concrete significant for serviceability conditions, f_{R1} residual strength of fiber reinforced concrete significant for ultimate conditions.

The value of $CMOD_{.}=2.5$ mm.

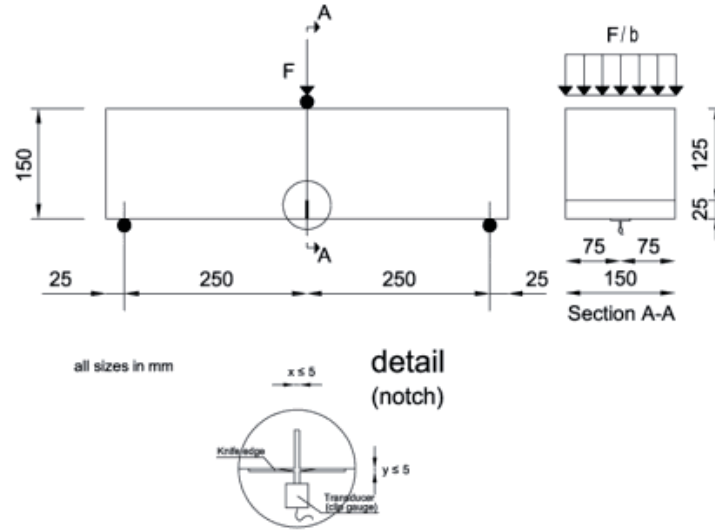


Fig. 1 Test set-up required

Beams without longitudinal and shear reinforcement

For SFRC members without both longitudinal and transverse reinforcement, the principal tensile stress s_1 , shall not be higher than the design tensile strength of steel fiber:

$$s_1 \leq f_{Ftuk} / \gamma_F \tag{2}$$

where: f_{Ftuk} [MPa] is the characteristic value of the ultimate residual tensile strength of FRC

γ_F is the partial safety factor for fiber steel.

The design value for the shear resistance in SFRC members with conventional longitudinal reinforcement and without shear reinforcement is given by the formula:

$$V_{Rd,F} = \left\{ \frac{0,18}{\gamma_c} k [100 \rho_1 (1 + 7,5 \frac{f_{Ftuk}}{f_{ctk}}) f_{ck}]^{1/3} + 0,15 \sigma_{cp} \right\} b_w d (1 + x)^n \tag{3}$$

where:

γ_c is the partial safety factor for the concrete without fibers;

k is a factor that which takes into account the size effect and it is equal to:

$$1 + \sqrt{\frac{200}{2d}} \leq 2,0 \tag{4}$$

d is the effective depth of the cross section, in mm

ρ_1 is the reinforcement ratio for longitudinal reinforcement equal to:

$$\rho_1 = A_{s1} / b_w d \tag{5}$$

A_{st} is the cross sectional area of the reinforcement which extends $\geq l_{bd}+d$ beyond the considered section; in mm^2 ;

f_{Ftk} is the characteristic value of the ultimate residual tensile strength for FRC, by considering $w_u = 1.5 \text{ mm}$ [MPa];

f_{ctk} is the characteristic value of the tensile strength for the concrete matrix, in MPa;

f_{ck} is the characteristic value of concrete cylindrical compressive strength, in MPa;

$s_{cp} = N_{Ed}/A_c < 0.2 f_{cd}$ [MPa] is the average stress acting on the concrete cross section, A_c [mm^2], for an axial force N_{Ed} [N], due to loading or prestressing force ($N_{Ed} > 0$ for compression);

b_w is the smallest width of the cross-section in the tensile area, in mm.

The shear resistance, $V_{Rd,F}$ is assumed to be not smaller than the minimum value, $V_{Rd,Fmin}$, defined as:

$$V_{Rd,Fmin} = (v_{min} + 0,15 s_{cp})$$

$$v_{min} = 0,035 k^{3/2} f_{ck}^{-1/2} \quad (6)$$

Beams with shear and longitudinal reinforcement

For the design of members with conventional shear reinforcement applies formula,

$$V_{Rd} = V_{Rdc} + V_{Rds} \quad (7)$$

In the case of FRC members this equation is extended to:

$$V_{Rd} = V_{Rdc} + V_{Rds} + V_{RdF} \quad (8)$$

where:

V_{RdF} - is the contribution of the steel fibers, intersecting the design shear crack.

Shear capacity of SFRC beams according to the Russian Code

Russian Code SP-52-104-2007 proposes the formulas to calculate the tensile strength of SFRC elements. When calculating the tensile strength, it is assumed that the tensile stresses are evenly distributed throughout the cross-section of the tension. Also, it is assumed that the fiber is uniformly distributed in a volume element.

A tensile strength of steel fiber reinforced concrete is denoted as f_{fct} .

For determination of f_{fct} strength there are two models of element destruction:

- The first model: the tension element is destroyed as a result of the interruption of certain fiber and pulling the rest:

$$l_{f,an} < \frac{l}{2} \quad (9)$$

The second type: the tension element is destroyed, assuming that all the fibers

have been drawn from concrete

$$l_{f,an} \geq \frac{l_f}{2} \quad (10)$$

where $l_{f,an}$ is anchorage length

If it is the first element the mode of failure, use the formula

$$f_{fct}^f R_{fct} = m_1 \left[K_T k_{or}^2 \mu_{fv} R_f \left(1 - \frac{l_{f,an}}{l_f}\right) + 0,1 R_b (0,8 - \sqrt{2\mu_{fv} - 0,005}) \right] \quad (11)$$

where:

m_1 – coefficient depending on the work of fibers in concrete,
 k_{or} – coefficient taking into account fiber orientations of in a volume element, depending on the dimensions of element cross section and length of the fibers, acc. taken from the standard SP-52-104-2007

μ_{fv} – ratio of the ratio of fibers in the concrete volume (v);

K_T – coefficient calculated using the formula:

$$K_T = \sqrt{1 - (1,2 - 80\mu_{fv})^2} \quad (12)$$

If there is the second model of damage element, it should be used the following formula:

$$f_{fct}^f R_{fct} = m_2 R_b \left(K_T \frac{k_{or}^2 \mu_{fv} l_f}{8\eta_f d_{f,red}} + 0,8 - 0,5\mu_{fv} \right) \quad (13)$$

where:

m_2 – coefficient depending on the work of fibers in concrete

The coefficients m_1 and m_2 for engineering structures are based on experimental studies.

Shear model for SFRC concrete elements

Shear load capacity of steel fiber-reinforced concrete members with combined reinforcement (steel fibers and stirrups) may be defined on the basis of model diagonal cross-sections, presented in Russian Code SP-52-104-2007. The presented model is based on the equilibrium conditions of forces and bending moments in the inclined sections created by the inclined crack in the shear zone (see Fig.2 and 3).

Calculation of shearing strength must be checked separately considering the effect of bending moment and shear force. Strength diagonal cross-sections determined by the equations of equilibrium of external and internal forces and bending moments acting on the assumed cross section. Design shear force V_{Ed} is defined in cross-section perpendicular to the longitudinal axis and design moment M_{Ed} is obtained from the summary of the moments of all the design forces to the center of gravity of compression zone.

Calculation of shear capacity V_{Rd} for steel fiber-reinforced concrete elements without stirrups in the zone of inclined crack (Fig.2) in the beam is based on the formula

$$V_{Rd} = V_{fcc} + V_{fct} \quad (14)$$

where: V_{Rd} - shear capacity in a cross-section perpendicular to the longitudinal axis in the element, V_{fcc} - shear force, carried by compression zone of fiber-reinforced concrete in this section; V_{fct} - shear force, carried by tensile zone of steel fiber-reinforced concrete in the section.

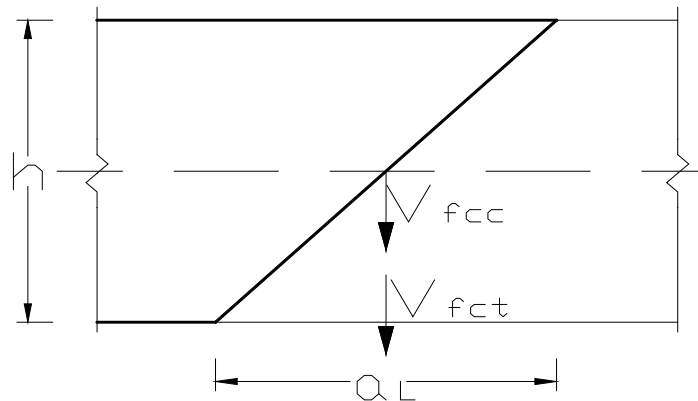


Fig. 2. Superposition of shear force in the fiber-reinforced concrete beam without stirrups (SP-52-104-2007)

The value of shear force V_{fcc} for flexural members may be determined by the formula:

$$V_{fcc} = \frac{0,75 f_{fct} b h^2}{a_L \sin(90^\circ - \beta)} \quad (15)$$

where: f_{fct} - defined by the formulas with the substitution of two coefficients k_{or} or $k_{n,w}$
 k_{or} - coefficient taking into account the orientation of the fibers relative to the direction of principal tensile stresses; $k_{n,w}$ - coefficient taking into account the influence of the orientation of the fibers sectional dimensions perpendicular to the direction of an external force, b and h - the width and height of cross-section of the calculated member; a_L - the projection of an inclined crack (the angle is assumed to be 45°); β - angle of the vertical axis of element to the vertical.

The value of shear force V_{fct} is given by:

$$V_{fct} = q_{fcc} \cdot a_L \quad (16)$$

where:

$$q_{fcc} = \frac{f_{fcc} b}{a_L \sin(90^\circ - \beta)} \quad (17)$$

f_{fcc} is the characteristic value of compressive strength steel fiber-reinforced concrete

Calculation of shear capacity V_{Rd} for fiber-reinforced concrete members with flexural reinforcement (stirrups), see Fig.3, is based on the formula

$$V_{Rd} = V_{fcc} + V_{fct} + V_{sw} \quad (18)$$

where: V_{sw} - shear force, carried by the shear reinforcement in the cross-section.

Transverse force taken by the stirrups V_{sw} is determined by the formula

$$V_{sw} = \varphi_{sw} \cdot q_{sw} \cdot a \quad (19)$$

where:

φ_{sw} - a coefficient taken equal to 0.75;

q_{sw} - force in the transverse reinforcement per unit length of the element, determined by the formula:

$$q_{sw} = \frac{f_{sw} A_{sw}}{s_w} \quad (20)$$

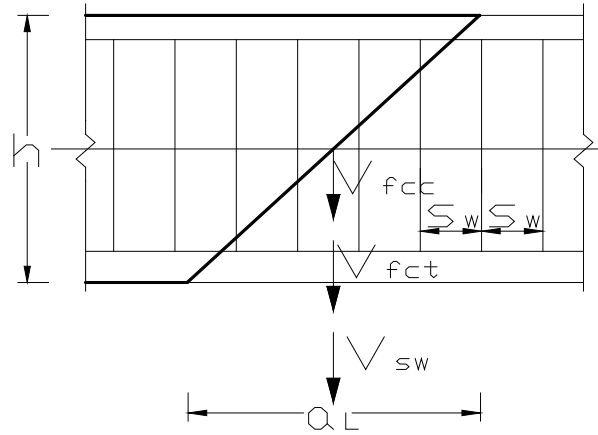


Fig. 3. Composition of shear forces in the fiber-reinforced concrete beam having flexural and shear reinforcement

Calculation of shear capacity of SFRC beams on the basis of experimental studies

Lim and Oh (Lim, Oh 1999) conducted the first tests on the shear capacity of flexural beams made of steel fiber reinforced concrete. Theoretical and experimental research has been carried out on 9 beams by testing varied fraction of steel fibers and the ratios of steel stirrups required to shear reinforced. On this basis they formulated the formula for the total shear force for the simply supported beams:

$$V = V_c + V_A + V_d + V_f \quad (21)$$

where V_c is the shearing force across the compression zone, V_a is the interlocking aggregate force, V_d is the dowel action force, V_f is the vertical components of the fiber pull out force along the inclined crack, as showed in Fig 4.

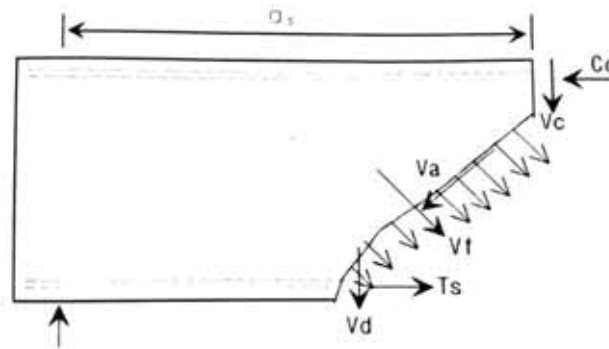


Fig.4. A part of shear span of a simply supported SFRC beam (Lim, Oh 1998)

Recent studies on the effect of fiber on shear capacity of the fiber are presented by Aouda, and others (Aoude, et al. 2012). Experimental program involving 9 series of RC and SFRC beams. Investigated shear capacity, failure mode and crack openings. Based on performed research there was carried out theoretical research to establish the formula for the shear capacity considering the effect of fibers. Presented model lead to equation (22) where the shear resistance of an SFRC beam was assumed to be equivalent to the expected shear strength of traditional RC beam V_{no} plus the additional shear resistance V_{fib} provided by the fibers.

$$V_{nf} = V_{no} + V_{fib} \quad (22)$$

Shear resistance V_{no} - is general design method of CSA A23.3-04 showed in equation (23).

$$V_{no} = V_c + V_s \quad (23)$$

The V_c and V_s are the concrete and transverse steel (stirrup) contributions to shear resistance.

The fiber contribution the shear resistance V_{fib} can be related to the pullout strength F_p of the fibers crossing this cracking plane, as shown in eq. (24).

$$V_{fib} = (N_{fib} 0,83 F_p) b_w d_v \cot \theta \quad (24)$$

The effective number of fibers per unit area N_{fib} for fibers randomly oriented in three dimensions can be calculated using eq. (25)

$$N_{fib} = V_f / A_f \alpha \eta_l \quad (25)$$

where A_f is the cross-sectional area of the fiber, v_f - the volume fraction of fibers in the matrix, α - the orientation factor and η_l - the length factor.

Own experimental investigation on shear capacity of SFRC

Team of Bialystok University of Technology conducted a study in order to clarify the possibility of partial or total replacement of stirrups with steel fibers in flexural model beams. The beams were performed of concrete mix using steel fibers with a length of 50 mm and a diameter of 1 mm made of round wire, cold drawn, low carbon steel. Steel fibers were dispensed to the concrete mix in an amount of 1.5% by volume of concrete. For preparation of SFRC model beams Portland cement CEM I 32.5 R and natural aggregates, having grain size of 0.125-4 mm. To improve the workability of the composite mixture a superplasticizer was used. In the mix water to cement ratio was equal to 0.67. Single-span reinforced concrete beams with a cross section 80 mm × 120 mm and the span 1,100 mm were loaded till failure. The reinforcing flexural steel bars are made of steel with a yield strength f_y of 500 MPa, and the steel of stirrups have the yield strength f_y of 220 MPa. The details of SFRC model beams are presented in Fig. 5. Depending on steel fiber content and arrangement of steel stirrups the 3 series of model SFRC beams have been performed containing three samples in each series (Iwaniuk, 2012). In any series, the spacing of steel stirrups was differed at the left and right half span of each model beam (as shown in Fig. 5).

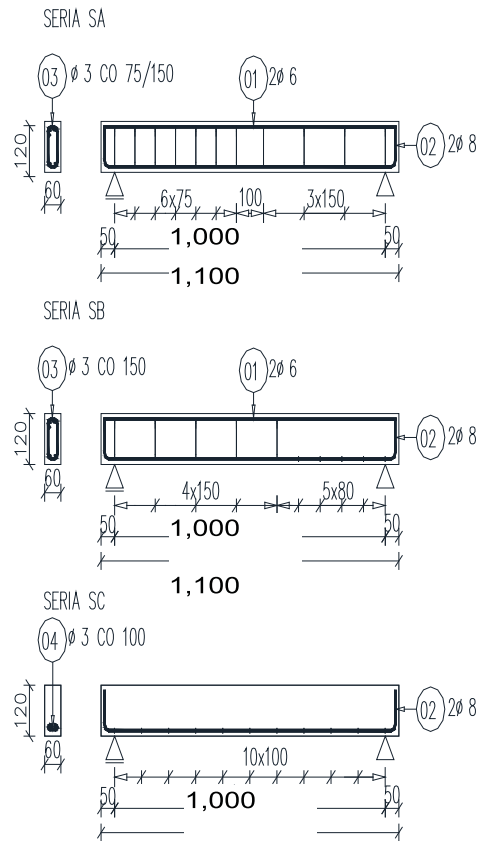


Fig. 5 Details of three Series of tested SFRC beams: SA, SB and SC (Iwaniuk, 2012)

Eight of the nine tested SFRC model beams were failed in shear. Only one beam has failed due to flexure. Fig. 6 shows the destructive diagonal cracks in the tested beams. The failures caused by the shear force occurring on this side of the beam, where the stirrups was used at twice increased spacing of stirrups or they were not used at all.



Fig. 6 The failure modes of selected SFRC beams after testing (Iwaniuk, 2012)

The test results obtained for SFRC model beams are presented in Table 2. The tested model beams with steel fibers for all the test series showed an increase shear capacity by about 45%.

Tab. 1 Critical forces for each series of model SFRC beams (Iwaniuk, 2012)

Series of beams	Maximum critical experimental load	Experimental critical shear capacity	Increase of theoretical shear capacity
	(kN)	(kN)	(%)
SA-0	24	12	-
SA-1	34*fail in bending	17	42.0
SA-2	34	17	42.0
SB-0	22	11	-
SB-1	32	16	45.0
SB-2	32	16	45.0
SC-0	20	10	-
SC-1	28	28	40.0
SC-2	30	30	50.0

Comparison of experimental results and theoretical

The calculation of the effect of longitudinal forces will be carried out in accordance with PN-EN 1992.

Tab. 2. Load theoretical and experimental beams of model (Iwaniuk, 2012)

Capacity on	Series	Theoretical destructive force [kN]	Destructive force of experimental studies [kN]	Increase in shear capacity for experimental research [%]
Bending	A, B, C	17	17*	-
	A	12	17**	42,0
Shear	B	10	16**	60,0
	C	7	15**	114,0

The values of critical shear forces evaluated from the tests for each series of test were significantly higher than the calculated theoretical shear capacities. The largest increase was obtained for beams of series C, where there was no stirrups.

Summary and conclusions

Model Code 2010 proposes separately the methods of calculation of shear capacity for beams with or without shear and longitudinal reinforcement. In the case without shear reinforcement fibers supplement is taken into account by determining the characteristic value of the residual ultimate tensile strength of FRC in accordance with the principles of fracture mechanics. For elements with steel shear reinforcement and longitudinal fibers also are included as a contribution of the fibers increasing the design shear cracks.

Russian standard specifies shear capacity of SFRC beams taking into account the impact of cross-diagonal dividing the fiber in tension and compression zones. The standard calculation formulas are presented in function of strength of fiber concrete in tension, taking into account the amount, type, length and diameter of the fibers. Unfortunately, these models are suitable only for fibers produced in Russia.

In all the reporting methods of computing the impact of fiber on shear capacity included as a component of the shear force. In view of the many factors the Model Code 2010 method is the most accurate method. The parameters used in the calculation are universal and do not depend on the country of origin of fibers, as in the case of the Russian method. However, the method of calculation laid down in the Russian standard is much simpler.

From own research done on model SFRC beams with and without stirrups shows that experimental values of destructive shear forces are much higher than theoretical ones.

For better understanding of the problem of determination of shear capacity of SFRC members a further experimental investigations of FRC beams are needed.

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