NUMERICAL PREDICTION OF WAVE REACTION OF THE LAYERED ELASTIC HALF-SPACE

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Summary:
This paper presents the results of numerical analysis obtained for layered elastic half-space loaded dynamically. The main problem was to study the dynamic response of the medium in conditions of multiple reflections and refractions of waves on the borders of layers.
The subject of this study was the dynamic behaviour of heterogeneous, isotropic, horizontally – layered, elastic half-space. The numerical calculations program included variations of the foundation. The foundation was composed of two layers, wherein the first layer's thickness was 2, 4 or 6 m. Wave propagation modelling was performed using the method of finite differences.
The decline of the amplitude of vertical vibrations along with the distance in uniform media generally is supported by the standardized prediction. The standardized prediction for the decline of amplitudes of the horizontal vibration along with the distance may lead to understating. Local fading and strengthening of amplitude of vertical and horizontal displacements occur. This effect should not necessarily be connected with ground medium stratification, even though such effects in case of stratified medium have increased. If a vulnerable layer is on top of the rigid one, larger amplitude of vertical displacement is obtained at the surface rather than in a uniform, weak medium. In the case of inverse layout of the layers, i.e. if the rigid layer is lying on top of a thick, vulnerable layer, the amplitude of vertical displacements on the surface are smaller compared to the amplitude obtained in a uniform, rigid medium.

Keywords: wave mechanics, vibrations, layered ground medium

Introduction

Determination of dynamic influences, which may affect a construction designed in the vicinity of the source of vibration, requires forecasting of wave propagation in the ground medium. Prediction can be based on a theoretical analysis, experimental studies or by using models that estimate the amplitude of vibration that may occur at the location of the object. Unquestionably the greatest confidence should be put in the results of experimental studies. The correctness of the theoretical predictions depends on simplifying assumptions about the geometric and physical modelling of the ground medium, as well as on the accuracy of the methods used, which almost always are numeric or numerical - analytical. Ready analytical formulas should be taken with some caution due to their unspecified conditions of accuracy and limitations of the scope of applicability. Formulas for estimating the amplitudes of vertical and horizontal displacements are discussed in the monograph by (Lipinski 1985). One of them is called Barkan curves. They were verified by experiments widely recounted in the work of (Ciesielski et alia (1996, 1997, 1999, Kwiecien 1997). They are also used in the standard PN-80/B-03040.

This paper presents the results obtained from the numerical analysis carried out for the ground medium treated as an isotropic elastic half-space with the heterogeneity of the layer type. Stratification is a natural feature of the ground medium and the corresponding geometric schematization model creates a heterogeneous medium. The medium layers may be positioned horizontally or tilted down, be in regular arrangements, or be folded in varying degrees. The so called „wedging" of the layers has also been observed. The occurrence of ground water also requires a separation of different geotechnical layers in the ground. In addition, a significant problem is the modelling of the building structure in terms of its interaction with the subsoil (example diagram in Figure 1). It is important here to apply appropriate contact zone at the interface between two media with significantly different mechanical properties. The elements of this interaction, however, are not the subject of analysis presented in the work.

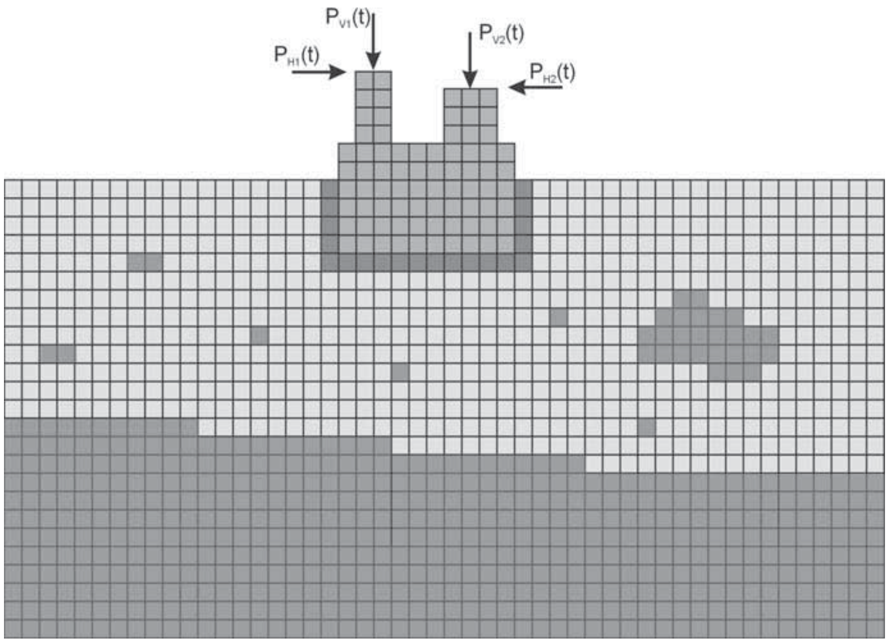

Fig. 1. Block foundation in diversified ground (cross section of spatial task)

Inclusion of stratification in geometric modelling of ground medium allows for a theoretical approach to characteristic effects of reflection and refraction of stress waves at the border of layers, testing whether any of the layers does not become a waveguide. These effects cause the instantaneous stress distributions to become highly unstable and significantly alter the stress – deflection response of the layered medium in relation to the uniform medium.

The results presented in the paper ware discussed at the 2nd Geotechnical Conference "Soil-Structure Interaction", Białystok - Białowieża 2004 (Bąk, Gosk 2004). Extended analysis based on that work are presented in this article.

Assumptions and basics of numerical analysis

The subject of this study was the dynamic behaviour of heterogeneous, isotropic, horizontally – layered, elastic half-space. The analysis covered a quarter of the half-space, which was approximated by a cuboids' area of a considerable size: $L_x = L_y = 100$ m, $L_z = 50$ m and the vertical planes of symmetry (x, 0, z), (0, y, z). The planes separating the area out of half-space included supports „transmitting" both, longitudinal and transverse waves. Also, harmonic, uniformly distributed load acting on a limited area of a rectangle with dimensions lx and ly, located as coordinates: $(\langle 0, l_x \rangle, \langle 0, l_y \rangle, z = 0)$ was taken into consideration. Numerical analysis was carried out on the basis of author's own calculation program which uses a differential algorithm proposed in a book by (Szczesniak 1999) for the spatial task of wave mechanics. This algorithm can automatically model the effects of multiple reflections and refractions of stress waves. There is no need of a separate tracking of forming of wave fronts. In the case of one-dimensional spatial tasks an error-free approximation of the problem of reflection and refraction of waves can be achieved, as in (Bak, Szczesniak, 1987).

Numerical analysis program

The numerical calculations program included variations of the foundation. Homogeneous areas and horizontally layered areas were studied . The foundation was composed of two layers, wherein the first layer's thickness was 2, 4 or 6 m. Analyzed variants of stratification are shown in Figure 2. The material constants reflect the dynamic characteristics of the soil. Applying harmonic load acting on a limited area of the medium is being considered.

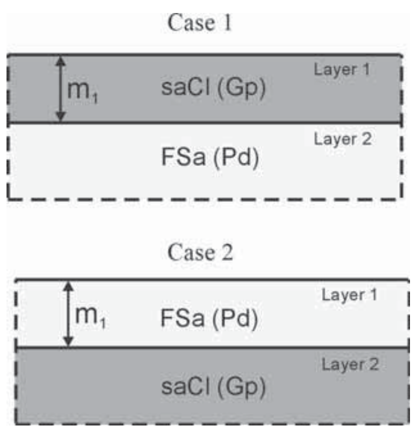

Fig. 2. Analysed ground variants

Sandy clay – saCl (Gp):

$$\rho = 2050\ kg/m^3 \qquad E = 320\ MPa \qquad v = 0.30 \qquad a_1 = 458\ \frac{m}{s} \qquad a_2 = 245\ \frac{m}{s}$$

Fine sand – FSa (Pd):

$$\rho = 1800\ kg/m^3 \qquad E = 120\ MPa \qquad v = 0.25 \qquad a_1 = 283\ \frac{m}{s} \qquad a_2 = 163\ \frac{m}{s}$$

$$m_1 = 2\ m \qquad m_1 = 4\ m \qquad m_1 = 6\ m$$

Studying of effects of reflections and refractions on the borders of layers

In general, in the case of the effects of reflection and refraction of a monochrome wave - angles, the amplitudes of the waves and shearing can be designated as an extended solution of algebraic equations. This system is created by the conditions for compatibility of displacements and stresses on the boundary of layers and Snell's law. In the case of a plane wave of amplitude A, incident perpendicularly to the plane of the boundary layers of a medium with varying impedance $(a\ \rho)_1 \neq (a\ \rho)_2$, and according to the work edited by (Kaliski 1966) the following results obtained:

$$A_0 = \frac{(a\rho)_2 - (a\rho)_1}{(a\rho)_1 + (a\rho)_2} A, \qquad A_z = \frac{2(a\rho)_1}{(a\rho)_1 + (a\rho)_2} A \tag{1}$$

The correctness of numerical analyses of the effects of reflection and refraction of a plane wave with a strong discontinuity at the front, was verified by authors - developed calculation program. The results for both cases of stratification of the ground are shown in Figure 3.

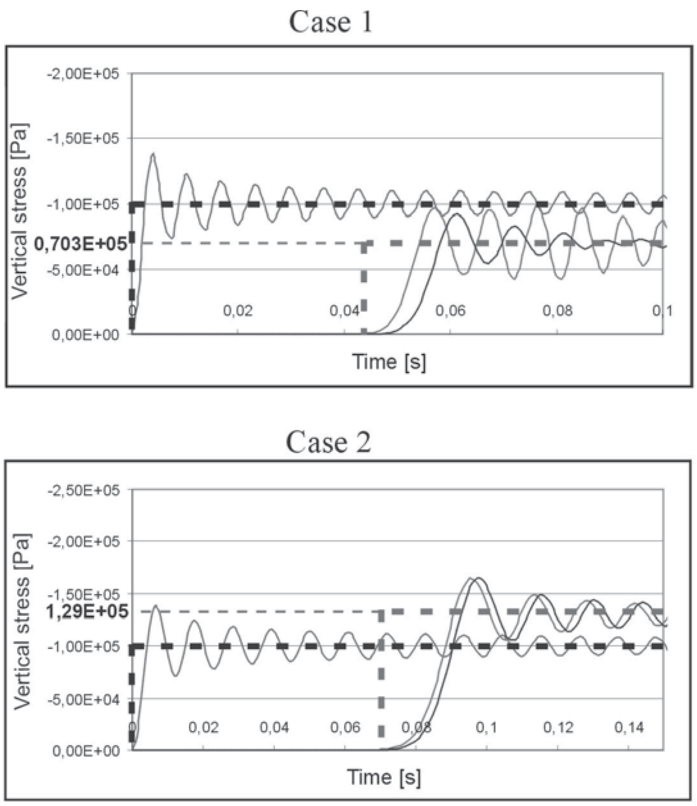

Fig. 3. Comparison of numerical results and theoretical solution

They are generally consistent with the theoretical prediction (1). A strong front of the stress wave, however, "vanishes in time", because of its inability to meet the condition of an error-free differential approximation.

Selected results of numerical analysis

The following are some of the results of the numerical analyses of the stratified and uniform ground displacement amplitudes.

Figures 4 and 5 show the boundaries of vertical and horizontal displacements on the surface of the ground with different stratification, but in the system: the upper layer, vulnerable – FSa (Pd) - lower, rigid – saCl (Gp). Highlighted, solid line matches the estimate of the amplitude of vibration with distance according to PN-80/B-03040. It is confirmed that it is correct in the uniform medium. Decline of amplitudes in layered media is monotonic. Another quantitative conclusion, constituting a direct consequence of the effects of reflection and refraction, is that the rigid, bottom layer of the foundation – saCl (Gp), increases the value of the amplitude, which is a result opposite to the experience from the static effects.

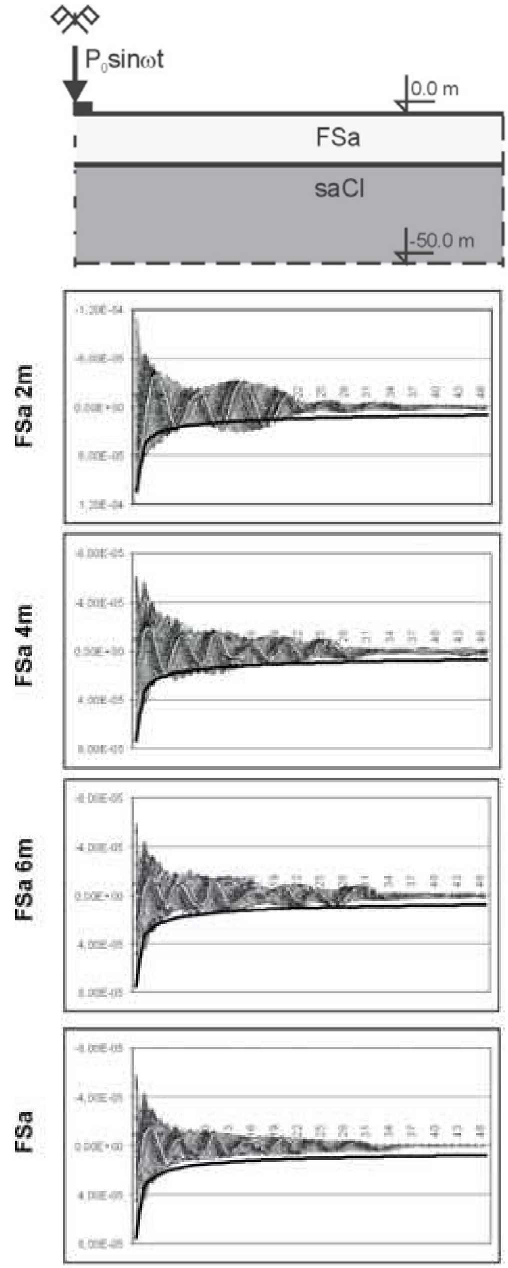

Fig.4. Vertical displacements boundaries [m] on the ground surface for various layers of ground medium
(n = 30Hz, thick line – prediction in accordance with the norm: PN-80/B-03040)

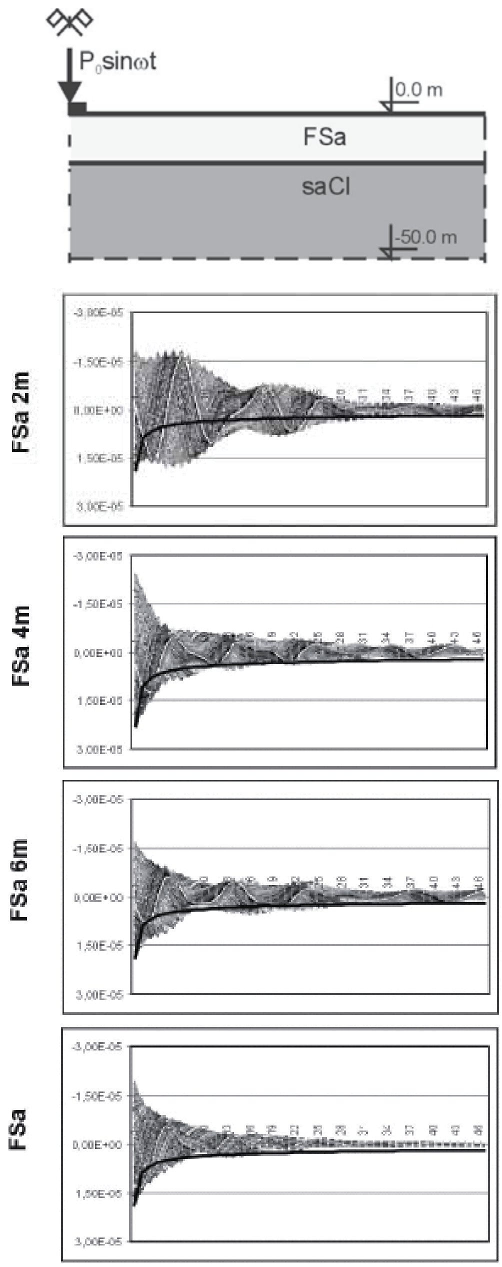

Fig.5. Horizontal displacements boundaries [m] on the ground surface for various layers of ground medium
(n = 30Hz, thick line – prediction in accordance with the norm: PN-80/B-03040)

Figure 6 shows the boundaries of horizontal displacement along the depth set out in points distant from the source by 12.5 m. Appropriate prediction of amplitude decline of these oscillations, which are considered as a result of the dominance of Rayleigh waves in deforming reaction of subsurface zone of the medium, taken from the work by (Lipinski 1985), can be represented as follows:

$$A_H(z) = A_H(0)\, e^{-\frac{2\pi z}{\lambda}\beta}, \tag{2}$$

and: z - ordinate along the depth, l - wavelength, b - a correction factor introduced for the purposes of satisfying comparisons made in this study, see Figure 6. The research recounted in the work by (Ciesielski et al 1997) showed that b should also cover the impact of dynamic load frequency.

Formula (2) proves that it can be observed that the correct prediction of decline of horizontal vibrations is only possible in subsurface ground medium, i.e. in the depth of shallow foundations. In the case of deep foundations one must take into consideration extremely different values of the amplitudes of horizontal displacement, which is a forced kinematic displacement of the erected object.

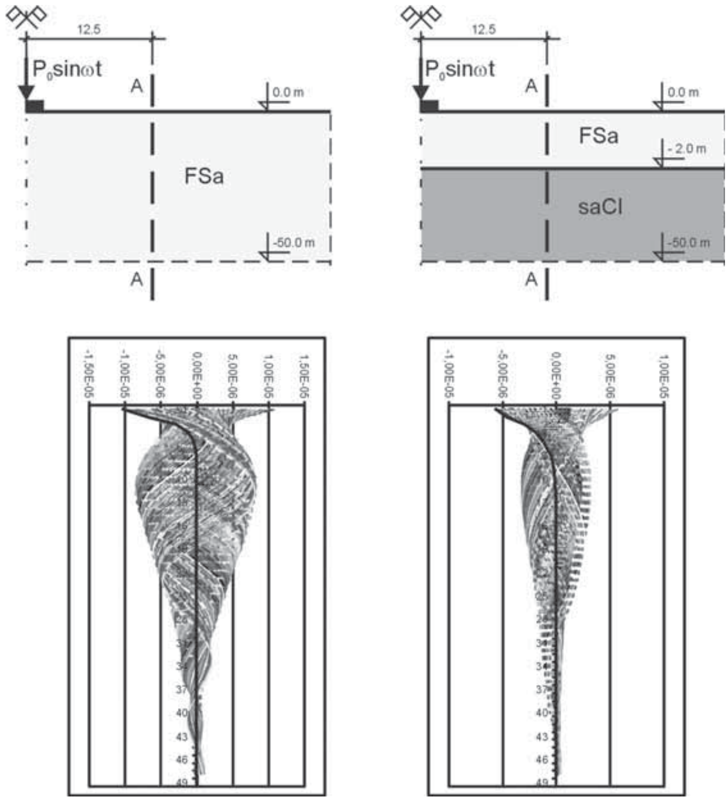

Fig.6. Influence of layered structure of the medium on horizontal displacements in central point of external impacts activity
(n = 10Hz, thick line –Barkan's prediction)

Shapes of maximum amplitudes of horizontal boundaries indicate that the range of relatively extensive horizontal vibration along the depth is significant, even in a uniform medium. It is not, however, in agreement with the conclusion relating to fast decline and depth of these vibrations. These conclusions are generally formulated on the basis of Rayleigh solution applied to a single wave. Only the prediction relating to the occurrence of a significant weakening of these vibrations right under the ground surface is applicable. However, at greater depths strong reinforcement covering a large area occurs. The rigid layer – saCl (Gp) under a vulnerable layer – FSa (Pd), significantly reduces the amplitude of horizontal displacement.

Conclusions

From the presented numerical results the following conclusions can be drawn:

- The decline of the amplitude of vertical vibrations along with the distance in uniform media generally is supported by the standardized prediction.
- The standardized prediction for the decline of amplitudes of the horizontal vibration along with the distance may lead to understating.
- Local fading and strengthening of amplitude of vertical and horizontal displacements occur. This effect should not necessarily be connected with ground medium stratification, even though such effects in case of stratified medium have increased.
- If a vulnerable layer is on top of the rigid one, larger amplitude of vertical displacement is obtained at the surface rather than in a uniform, weak medium - the effect of a weak layer being intensively compressed by stress interfering waves, incident and reflected. This was clearly observed in case of the FSa (Pd) layer with a thickness of 2 m.
- In the case of inverse layout of the layers, i.e. if the rigid layer is lying on top of a thick, vulnerable layer, the amplitude of vertical displacements on the surface are smaller compared to the amplitude obtained in a uniform, rigid medium - reflection effect - the amplitude of the reflected wave is negative, $(1)_1$.

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