SECTIONAL DESIGN OF R/C MEMBERS USING DEFORMATIONAL MODEL WITH INCLINED TOP BRANCH OF STEEL STRESS-STRAIN RELATIONSHIP

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Summary:
The paper presents the analysis of advanced cross-sectional design models of R/C flexural members using inclined top branch of stress-strain relationship for flexural reinforcing steel. The analysis was performed having considered more effective way to use realistic properties of ductility of flexural steel. Workflow is based on a basic assumptions described in the Eurocode 2 and also in Model Code 2010.
For concrete in compression the two models were considered: parabola-rectangle relationship and bi-linear stress-strain relation and for reinforcing steel - inclined and horizontal top branch models.
There were derived and presented formulas for evaluation of cross-sectional parameters for the members with rectangular cross-section shape using the parabola-rectangle and bi-linear stress-strain relation for all the concrete classes and steel with inclined branch of strains. Adaptation in design of analytical formulas is very labour-consuming, therefore cross-sectional parameters for normal concrete and HSC were written using tabular coefficients and presented in the tables. As calculation examples of dimensioning of flexural beam are the areas of flexural reinforcement are compared for member with rectangular cross-section.

Keywords: stress-strain relationship, reinforced concrete, cross-sectional design, ductility of steel.

Introduction

The evaluation of flexural capacity of reinforced concrete members according to recent codes like EN 1992-1-1 (2004) and Model Code 2010 (2012) is based on assumptions of a non-linear stress-strain relationship for concrete in the compressive zone and bi-linear relationship representing steel behavior under loading till failure (Ultimate Limit State).

The exact formulas applied for computation of the ultimate bending moment, which can be resisted by the reinforced beams using deformational model are much more complex in comparison to simplified model using rectangular stress block, due to necessity of possible true approximation of concrete behavior in compression zone. Any approximation induces various parameters to the function representing the stress distribution in concrete compressive zone, which is strongly dependent from the stress-strain diagrams. Codes usually are providing certain parameters, which should be applied for the estimation of the stress distribution in compressive zone.

The paper presents the comparison of results of computations of the ultimate bending capacity for flexural members using non-linear model for concrete and bi-linear model for reinforcing steel with the inclined branch. For estimation of the resultant of stresses in compressive concrete zone different values of characteristics of the strain functions (e_{c2} , e_{c2u}, n, parameters of limit strain and exponent of stress – strain function respectively) are applied following European Standard EN 19921-1 (2004) and Model Code 2010 (2012). This comparison shows that the assumed and applied in different standards strain coefficients play a significant role on the magnitude of the computed ultimate flexural capacity.

Theoretical approach for flexural members

Functional stress-strain relationships for concrete in compression

Parabolic-rectangular compressive stress distribution shown in Fig.1a for ultimate flexural limit state design of reinforced concrete sections is recommended by recent European structural concrete codes (Eurocode 2 and Model code 2010). The three strain characteristics: n, $\varepsilon_{c2,}$ ε_{cu2} defining the distribution of stresses in compression zone and recommended in standards are indicated and explained in the Fig.1a. The distribution of stresses in compression zone of considered section under bending is given by the formulas:

When the concrete strains are in the range $0 \le \varepsilon_c \le \varepsilon_{c2}$

$$\sigma_c = f_d \left[1 - \left(1 - \frac{\varepsilon_c}{\varepsilon_{c2}} \right)^n \right] \tag{1}$$

and for the range $\varepsilon_{c2} \le \varepsilon_c \le \varepsilon_{cu2}$

$$\sigma_c = f_d \tag{2}$$

where: f_{cd} – design compressive strength for given concrete class,

ε_{c2} , ε_{cu2}, n – parameters of limit strain and exponent of stress – strain function respectively

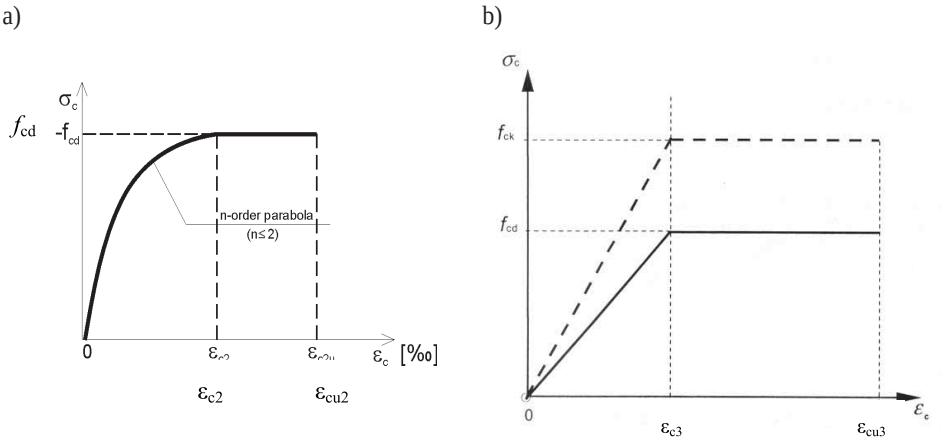


Fig.1. Stress - strain relationships for concrete used for ultimate limit state design (EC2 2004 Model Code 2010):

a) parabola-rectangle relation b) bi-linear relation

The three strain characteristics are: *n* - exponent of stress - strain function and two values of strains: ε_{c2} representing strain at the border between curved and rectangular part of the stress strain diagram, and ε_{cu2} representing ultimate strain at failure (see Fig.1a).

In cross-sectional analysis could be used also bi-linear relationship for stress-strain function for concrete in compression. In Fig.1b there is presented design diagram of compressive stresses and strains with limited values of strains ε_{c3} and ε_{cu3} (Eurocode 2, Model Code 2010).

Reinforcing steel properties

Informations about behaviour of reinforcing steel may be found in Eurocode 2 in subsection 3.2 and in annex C. The rules presented there have general character and describing basic requirements and qualities of the product, specified in European Standard (EN 10080 2006).

The deciding influence in designs have: yield strength (f_{yk} or $f_{0,2k}$), designed yield strength f_{yd} and idealized stress-strain relation, taking into account reinforcing steel ductility, depending on ultimate deformation ε_u and ratio of tensile strength to the yield stress $(f_t/f_y)_k$.

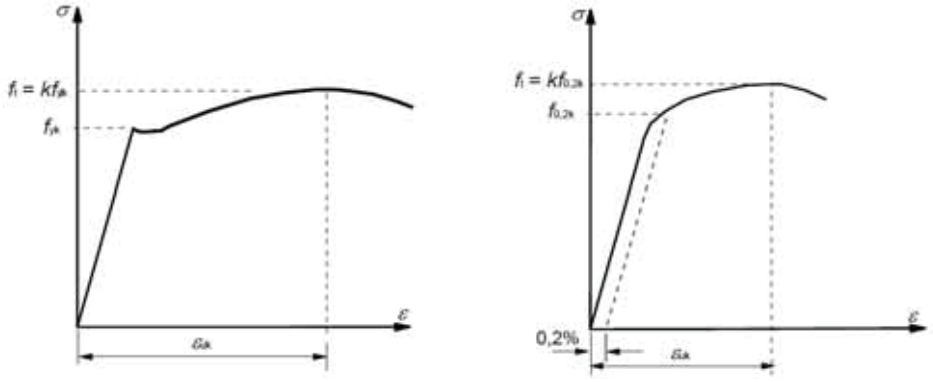
The application rules for design and detailings are specified in Eurocode 2. The yield strength f_{yk} ranges takes from 400 to 600MPa. In Poland the commonly used reinforcing steel has characteristic yield strength equal f_{yk} = 500MPa.

Table 1. Properties of steel used in Poland and theirs characteristics, according to Eurocode 2 (Strosolski 2011)

Steel class	Type	Yield strength [MPa]		Tensile strength f_t [MPa]	ε_{uk} [‰]	$k = (f_{tk}/f_{yk})$
		f_{yk}	f_{yd}			
A	B500A	500	435	min 525	>2,5	>1,05
A	RB500	500	435	min 550	>2,5	>1,05
A	RB500W	500	435	min 550	>2,5	>1,05
B	RB400	400	347	min 440	>5	>1,08
B	34GS	410	356	min 590	>5	>1,08
B	BSt500S(B)	500	435	min 550	>5	>1,08
C	B500SP	500	435	min 575	>8	1,15-1,35

Ductility is understand as large strain capability of reinforcement after reaching yield strength: „*Reinforcement ductility is defined by requirements on the elongation at maximum force* ε_{uk} *and by the ratio of tensile strenght to the yield stress*" (Knauff 2006, 2012). It is specified by the two parameters: k and ε_{uk} (see Fig.2). The value of k denotes

$$k = \left(\frac{f_t}{f_y} \right)_k \tag{3}$$



a) hot rolled steel b) cold worked steel

Fig. 2. Stress-strain diagrams of typical reinforcing steel (Eurocode 2, Model Code 2010)

In Table 2 there are presented properties of reinforcing steel, taken from Eurocode 2. It should be noticed, that k and ε_{uk} factors are given as a minimum values. It may be used when we don't have exact steel specifications given in certificates provided by the manufacturer.

Table 2. Properties of reinforcement (Annex C toEurocode 2)

Property of steel	Bars and de-coiled rods			Wire Fabrics			Requirement of quantile value (%)
Class	A	B	C	A	B	C	-
Characteristic yield strength f_{yk} or $f_{0,2k}$ (MPa)	400 do 600						5,0
Minimum value of $k = (f_t/f_y)_k$	≥1,05	≥1,08	≥1,15 <1,35	≥1,05	≥1,08	≥1,15 <1,35	10,0
Characteristic strain at maximum force, ε_{uk} (%)	≥2,5	≥5,0	≥7,5	≥2,5	≥5,0	≥7,5	10,0

On the Polish market there are actually available many classes of high strength steels. However, only one of them is having the best ductility parameters: class B500SP with the EPSTAL quality mark. Material parameters of this steel allows to qualify it to class C (high ductility class). Today, most commonly used classes of steel are those with medium ductility.

For normal designs, Eurocode 2 allows to use one of the following assumptions (see Fig. 3):

a) an inclined top branch: strain limit equals ε_{ud} and a maximum stress of kf_{yk}/γ_s

b) a horizontal top branch: strain limit may not be checked.

The recommended value of ε_{ud} is 0,9 ε_{uk}.

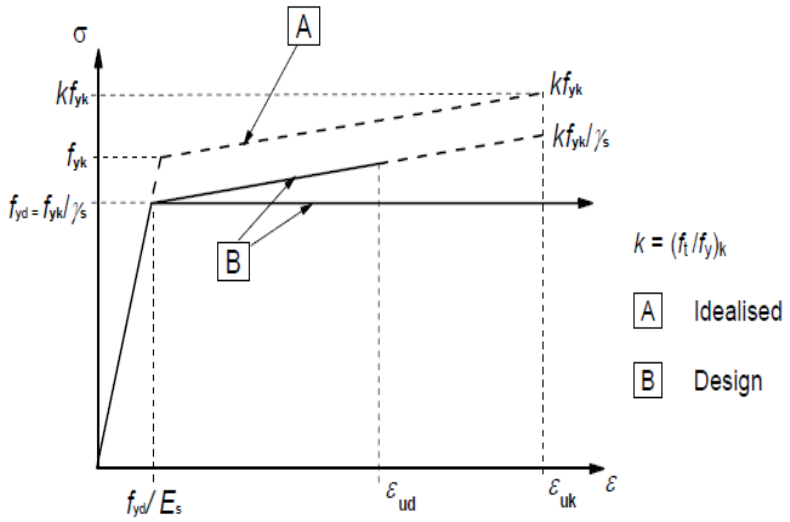


Fig. 3. Idealised (A) and design (B) stress-strain diagrams for reinforcing steel (Eurocode 2, 2004)

In model a) with inclined top branch, steel reaching yield strength at ultimate strain equals ε_{ud}. In b) model with horizontal top branch it was assumed that stress are constant and equal to design yield stress, and reinforcement strains are not limited. Commonly, design was based on model b) with horizontal top branch. Formerly, in both design models reinforcing steel strains ε_{uk} were limited for reinforcement bars to 10‰. There are no such limits in Eurocode 2.

In presented article will be analyzed model of steel with inclined top branch.

Analysis of cross-sectional forces for parabola-rectangular model for concrete

Main assumptions

According to Fig. 4 the external bending moment is balanced by following internal forces:

- resultant of compression stresses in concrete F_c
- resultant of stress in reinforcing steel tension F_s

Resultant of compression stress in concrete F_c may be computed as a volume of stress block, based on compression zone of the cross-section. The second parameter for volume evaluation depends of the shape of stress diagrams.

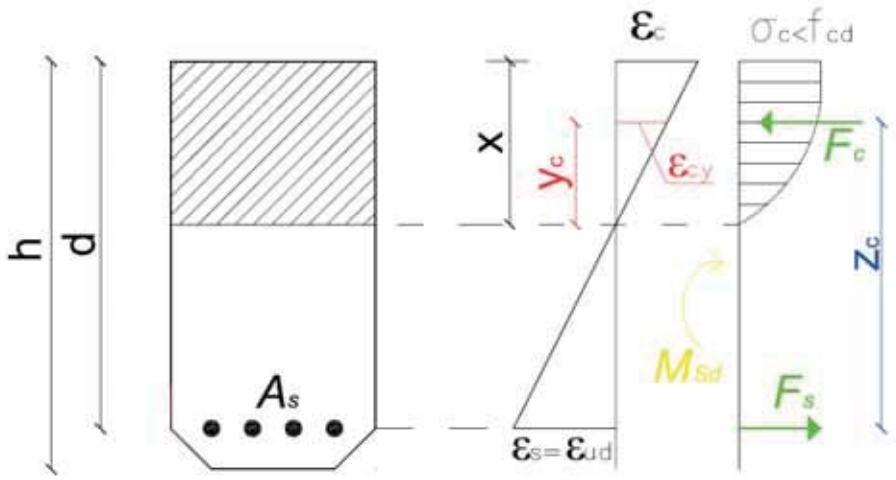


Fig. 4. Resultant of internal forces in any cross-section of flexural R/C member

For determination of internal forces moment M_{Rd} it is nessesary to previously define internal level arm z_c – distance between resultants of forces F_c and F_s (according to Fig. 4).

$$z_C = d - x + y_C \tag{4}$$

y_c - distance between centre of gravity of stress block and the neutral axis of the element.

Resultant of stresses in flexural reinforcement may be written in following form:

$$F_s = A_s \sigma_s \tag{5}$$

$\sigma_s = f_{yd}$ – for variant with horizontal top branch for reinforcing steel deformations.

In variant with inclined top branch for reinforcing steel deformations, value σ_s can be designated from proportion, in which first should be determined range of deformations in reinforcing steel.

Ultimate limit state is determined on the basis of internal bending moment (flexural capacity):

$$M_{Rd} = F_c z_c \tag{6}$$

While range of the compression zone x in singly reinforced cross-section is defined from equation of internal forces:

$$F_c - F_s = 0 \tag{7}$$

Considering cross-section of the element and its bearing, independent of its shape may be made following assumptions:

1. Plane-sectional law according to Bernulli's rule (cross-section deformations are proportional to theirs distance to the neutral axis).
2. Deformations of reinforcement and concrete surrounding are the same for both materials $\varepsilon_s = \varepsilon_{cs}$
3. Concrete tensile strength f_{ct} is ignored
4. Stresses in concrete in compression is derived from the design relationship σ_c-ε_c
5. Stresses in the reinforcing steel are derived from the design curve σ_s-ε_s

Based on above assumptions there were derived formulas for evaluation of cross-sectional parameters for rectangular shape of elements. These formulas are depending on the combination of strain domains in concrete compression zone and in tension flexural steel (Lapko 2001, Lapko Jensen 2006). Because of concrete compressive strain conditions, the shape of the stress diagrams in the compressed zone can be different. Therefore, compression stress block F_c will be described with different functions according to the range of strains in concrete in compression (Lapko Jensen 2006).

Table 3. Strain domains assumed in the analysis (Lapko and Jensen 2006)

Strain Domain	1a	1b	2
Strains in concrete ε_c	$\varepsilon_c \le \varepsilon_{c2}$	$\varepsilon_{c2} \le \varepsilon_c \le \varepsilon_{cu2}$	$\varepsilon_c = \varepsilon_{cu2}$
Strains in steel ε_s	$\varepsilon_s = \varepsilon_{ud}$	$\varepsilon_s = \varepsilon_{ud}$	$f_{yd}/E_s \le \varepsilon_s \le \varepsilon_{ud}$

Solutions for normal concrete classes for $f_{ck} \le 50$ MPa

Combination of strains in domain 1a (see Fig.5)

$$\varepsilon_c \le \varepsilon_{c2} = 2{,}0‰ \qquad \varepsilon_{cu2} = 3{,}5‰ \qquad n = 2 \qquad \varepsilon_{ud} = \varepsilon_{su}$$

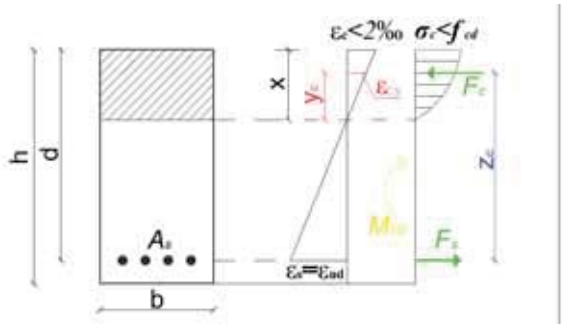


Fig. 5. Strain and stress diagrams in rectangular cross-section of flexural member in Domain 1a for normal concrete classes

The resultant of compressive stress block can be derived from the integral formula:

$$F_c = f_{cd} b \int_0^x \left[1 - \left(1 - \frac{\varepsilon_{cy}}{\varepsilon_{c2}} \right)^n \right] dy \tag{8}$$

Knowing, that $n=2$ we have:

$$F_c = f_{cd} b \int_0^x \left[1 - \left(1 - \frac{\varepsilon_{cy}}{2} \right)^2 \right] dy = f_{cd} b \int_0^x \left[\varepsilon_{cy} - \frac{{\varepsilon_{cy}}^2}{4} \right] dy \tag{9}$$

From linear proportions we obtain the value of ε_{cy} (see Fig.6)

$$\frac{\varepsilon_{cy}}{y} = \frac{\varepsilon_{su}}{d-x} \Rightarrow \varepsilon_{cy} = y \frac{\varepsilon_{su}}{d-x} \tag{10}$$

Substitution this value into the equation $\varepsilon_{cy} = y \dfrac{\varepsilon_{su}}{d-x}$ we obtain:

$$F_c = f_{cd} b \int_0^x \left[\varepsilon_{su} \frac{y}{d-x} - \left(\varepsilon_{su} \frac{y}{d-x} \right)^2 \frac{1}{4} \right] dy = f_{cd} b \left[\frac{\varepsilon_{su}}{2(d-x)} x^2 - \frac{{\varepsilon_{su}}^2}{(d-x)^2} \frac{x^3}{12} \right] \tag{11}$$

or for nondimensional value of neutral axis depth $\xi = x/d \Rightarrow x = \xi d$

$$F_c = f_{cd} b d \left[\frac{\xi^2 \left(\frac{1}{2} \varepsilon_{su} - \frac{1}{2} \varepsilon_{su} \xi - \frac{1}{12} {\varepsilon_{su}}^2 \xi \right)}{(1-\xi)^2} \right] \tag{12}$$

Internal lever arm can be obtained from the expression (see Fig.6):

$$z_c = d - x + y_c \quad \text{or} \quad z_c = d - \xi d + y_c \tag{13}$$

y_c- location of stress block centre of gravity in compression zone of the member:

$$y_c = \frac{\xi \left[\frac{1}{3} - \frac{1}{3} \xi - \frac{1}{16} \varepsilon_{su} \xi \right]}{\frac{1}{2} - \frac{1}{2} \xi - \frac{1}{12} \varepsilon_{su} \xi} d \tag{14}$$

Finally we get:

$$z_c = d\left[\frac{\frac{1}{2}+\xi\left(-\frac{2}{3}-\frac{1}{12}\varepsilon_{su}\right)+\xi^2\left(\frac{1}{6}+\frac{1}{48}\varepsilon_{su}\right)}{\frac{1}{2}-\frac{1}{2}\xi-\frac{1}{12}\varepsilon_{su}\xi}\right] \tag{15}$$

Internal bending moment is calculated as a product of resultant force F_c and lever arm z_c

$$M_{Rd} = F_c z_c \tag{16}$$

$$M_{Rd,1a} = f_{cd} b d^2 \left[\frac{\xi^2 \varepsilon_s \left[\frac{1}{2}+\xi\left(-\frac{2}{3}-\frac{1}{12}\varepsilon_s\right)+\xi^2\left(\frac{1}{6}+\frac{1}{48}\varepsilon_s\right)\right]}{(1-\xi)^2}\right] \tag{17}$$

Combination of strains in domain 1b

$\varepsilon_{c2} = 2{,}0‰ \le \varepsilon_c \le \varepsilon_{c2u} = 3{,}5‰$, $\varepsilon_{ud} = \varepsilon_{su}$

On the basis of above given assumption the following formulas were obtained in nondimensional value of neutral axis depth Resultant of compressive stresses is expressed as:

$$F_c = f_{cd} b d\left(\frac{3\varepsilon_{su}\xi + 2\xi - 2}{3\varepsilon_{su}}\right) \tag{18}$$

Internal lever arm z_c

$$z_c = d\left[1 - \frac{3\varepsilon_{su}\left[\frac{1}{2}\xi^2 - \frac{2}{3}\xi\frac{1-\xi}{\varepsilon_{su}} + \frac{1}{3}\frac{(1-\xi)^2}{\varepsilon_{su}{}^2}\right]}{3\varepsilon_{su}\xi + 2\xi - 2}\right] \tag{19}$$

Internal bending moment $M_{Rd,1b}$

$$M_{Rd,1b} = b d^2 f_{cd}\left[\frac{\xi^2\left(-2-\frac{3}{2}\varepsilon_{su}-\frac{1}{\varepsilon_{su}}\right)+\xi\left(3\varepsilon_{su}+4+\frac{2}{\varepsilon_{su}}\right)-2+\frac{1}{\varepsilon_{su}}}{3\varepsilon_{su}}\right] \tag{20}$$

Combination of strains in domain 2

for $\varepsilon_c = 3{,}5‰ \qquad \varepsilon_{ud} \le \varepsilon_{su}$

The cross-sectional parameters are expressed as follows

$$F_c = f_{cd} bd\left(\frac{17}{21}\xi\right) \tag{21}$$

$$z_c = d\left(1-\frac{9}{238}\xi\right) \tag{22}$$

$$M_{Rd,2} = bd^2 f_{cd}\left[\frac{17}{21}\xi\left(1-\frac{99}{238}\xi\right)\right] \tag{23}$$

In shorter way all the above formulas may be written in following form, using functional coefficients depending on nondimensional neutral axis depth and the strains in flexural steel:

$$F_c = \omega(\xi)\cdot bdf_{cd} \qquad z_C = \zeta(\xi)\cdot d \qquad M_{Rd} = \mu(\xi)\cdot bd^2 f_{cd} \tag{24}$$

For the needs of design of flexural member with the rectangular cross-section using deformational model with the inclined branch of strains in steel the tables were elaborated. In any case the stresses in flexural steel depends on steel strain and its ductility characteristic (see Fig.3)

$$\sigma_s = f_{yd} + \frac{f_{yd}(k-1)\left(\varepsilon_s - \frac{f_{yd}}{E_s}\right)}{\left(\varepsilon_{ud} - \frac{f_{yd}}{E_s}\right)} \tag{25}$$

where k is steel ductility ratio, expressed in the formula (3) and ε_s is steel strain depending on the neutral axis depth as follows

$$\varepsilon_s = \varepsilon_c \frac{(1-\xi)}{\xi} \tag{26}$$

Table 4. Table for dimensioning of flexural member with rectangular shape for inclined top branch of reinforcing steel type of B500SP (Epstal)

Strain domains	Coefficients				Strains [‰]		
	ξ	ω	ζ	μ	ε_c	ε_s	σ_s
Domain 1a	0,01	0,003	0,997	0,003	0,73	72	500
	0,02	0,011	0,993	0,011	1,47	72	500
limit 1a / 1b	0,027	0,018	0,990	0,018	2,00	72	500
Domain 1b	0,03	0,021	0,989	0,021	2,23	72	500
	0,04	0,031	0,984	0,031	3,00	72	500
limit 1b / 2	0,0463	0,037	0,981	0,037	3,50	72	500
Domain 2	0,05	0,041	0,979	0,040	3,50	66,50	495
	0,06	0,051	0,974	0,050	3,50	54,83	484
	0,07	0,061	0,969	0,060	3,50	46,50	476
	0,08	0,065	0,967	0,063	3,50	40,25	470
	0,1	0,081	0,958	0,078	3,50	31,50	462
	0,12	0,097	0,950	0,092	3,50	25,67	457
	0,14	0,113	0,942	0,107	3,50	21,50	453
	0,16	0,130	0,933	0,121	3,50	18,38	450
	0,18	0,146	0,925	0,135	3,50	15,94	448
	0,2	0,162	0,917	0,148	3,50	14,00	446
	0,22	0,178	0,908	0,162	3,50	12,41	445
	0,24	0,194	0,900	0,175	3,50	11,08	443
	0,26	0,210	0,892	0,188	3,50	9,96	442
	0,28	0,227	0,884	0,200	3,50	9,00	441
	0,3	0,243	0,875	0,213	3,50	8,17	441
	0,32	0,259	0,867	0,225	3,50	7,44	440
	0,34	0,275	0,859	0,236	3,50	6,79	439
	0,36	0,291	0,850	0,248	3,50	6,22	439
	0,38	0,308	0,842	0,259	3,50	5,71	438
	0,4	0,324	0,834	0,270	3,50	5,25	438
	0,42	0,340	0,825	0,281	3,50	4,83	438
	0,44	0,356	0,817	0,291	3,50	4,45	437
	0,46	0,372	0,809	0,301	3,50	4,11	437
	0,48	0,389	0,800	0,311	3,50	3,79	437
	0,5	0,405	0,792	0,321	3,50	3,50	436
	0,52	0,421	0,784	0,330	3,50	3,23	436
	0,54	0,437	0,775	0,339	3,50	2,98	436
	0,56	0,453	0,767	0,348	3,50	2,75	436
	0,58	0,470	0,759	0,356	3,50	2,53	435
	0,6	0,486	0,750	0,364	3,50	2,33	435
	0,62	0,502	0,742	0,372	3,50	2,15	435
limit domain 2	0,631	0,511	0,738	0,377	3,50	2,05	435

Stress-strain diagram for steel type B500SP, depending on range of the depth of neutral axis may be presented also in Fig.6.

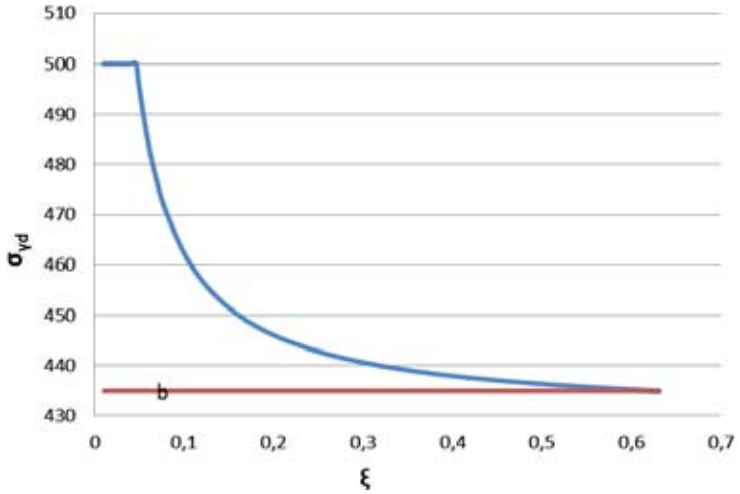


Fig. 6. Stress in steel versus neutral axis depth for B500P steel for the two models:
a) with inclined top branch, b) with horizontal top branch

Reinforcement ratios ρ diagrams in the cross-section of the flexural member depends on the depth of compression zone and concrete class for reinforcing steel are presented in Fig.7. Function $\rho(\xi)$ has been determined from the formula (32) using functional coefficients $\omega(\xi)$

$$\rho(\xi) = \omega(\xi)\frac{f_{cd}}{\sigma_s} \tag{27}$$

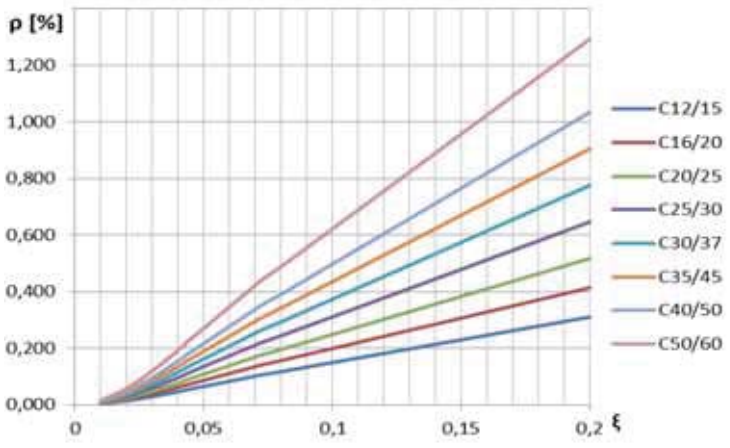


Fig. 7. Reinforcement ratios versus nondimensional neutral axis depth for given concrete classes

High Strenght Concrete classes

Algorithm used for calculation of assistance factors and dimensioning of rectangular cross-section of flexural R/C member made from High Strength Concrete (HSC) is analogous as for normal concrete classes (see p.3.2). However for this case that both limit concrete strain values ε_{c2} and ε_{cu2} are depending on HSC class. In the Eurocode 2 for highest concretes class C90/105 $\varepsilon_{c2} = \varepsilon cu_2$ = 2,6‰ and in the Model Code for class C 100/115 these values are equal to 2,7‰. Similarly, the values of *n* exponent of parabola function in formula (8) are depending on HSC class. Those formulas for cross-sectional parameters derived in paper of (Łapko Jensen 2006) are presented in Table 5.

Table 5. Overall method formulas, including variables strain of reinforcing steel in the flexural rectangular cross-section for high strength concrete (Lapko Jensen 2006)

Domain 1a	**Auxiliar parameters**
$F_c = [\xi + (1-\xi)pA]bdf_{cd}$	$p = \frac{\varepsilon_{su}}{\varepsilon_{c2}}$
$z_c = \left\{1 - \xi + \frac{0{,}5\xi^2 - (1-\xi)^2 (B-A)}{\xi + (1-\xi)pA}\right\}d$	$A = \frac{\left(1 - \frac{p\xi}{1-\xi}\right)^{n+1} - 1}{p^2(n+1)}$
$M_{Rd,1a} = \left\{(1-\xi)[\xi + (1-\xi)pA] + 0{,}5\xi^2 - (1-\xi)^2 (B-A)\right\}bd^2 f_{cd}$	$B = \frac{\left(1 - \frac{p\xi}{1-\xi}\right)^{n+2} - 1}{p^2(n+2)}$
Domain 1b	
$F_c = \left[\xi - \frac{1-\xi}{p(n+1)}\right]bdf_{cd}$	$p = \frac{\varepsilon_{su}}{\varepsilon_{c2}}$
$z_c = \left\{1 - \xi + \frac{0{,}5\xi^2 D - (1-\xi)^2 C}{\xi(D+1) - 1}\right\}d$	$C = \frac{1}{p(n+2)}$
$M_{Rd,1b} = \left\{\left(\xi - \frac{1-\xi}{D}\right)\left[1 - \xi + \frac{0{,}5\xi^2 D - (1-\xi)^2 C}{\xi(D+1) - 1}\right]\right\}bd^2 f_{cd}$	$D = p(n+1)$

Domain 2	
$F_c = \left\{ \xi \left[1 - \frac{1}{p_u (n+1)} \right] \right\} bdf_{cd}$	$p_u = \frac{\varepsilon_{c2u}}{\varepsilon_{c2}}$
$z_c = \left[1 - \xi + \xi \left(\frac{E-F}{2E-1} \right) \right] d$	$E = 0{,}5\, p_u (n+1)$
$M_{Rd,2} = \left[\xi \left(1 - \frac{1}{2E} \right) \left(1 - \xi \frac{E+F-1}{2E-1} \right) bd^2 f_{cd} \right]$	$F = \frac{1}{p_u (n+2)}$

Bi-linear stress-strain relations for concrete

The procedures for cross-sectional design using bi-linear stress-strain relationship for concrete and bi-linear relationship for steel are analogous to previous case (see Fig.1b). The combinations of strain limits and stress diagrams are presented in Fig.8.

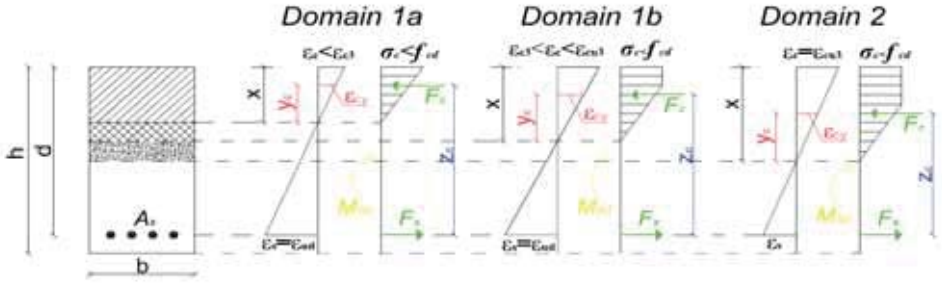


Fig. 8. Strains and stress diagrams for bi-linear model for concrete in strain domains 1a, 1b, 2.

From this analysis, there were derived the formulas, both for normal strength and High Strength Concrete classes. For normal concrete classes (for $f_{ck} \leq 50$ MPa) the formulas for cross-sectional parameters are listed in Table 6 and for HSC classes – are given in Table 7.

Table 6. Cross-sectional parameters derived using bi-linear model for normal strength concrete

Domain 1a	Domain 1b	Domain 2
$F_c = \frac{1}{2}\xi dbf_{cd}$	$F_c = \left(\frac{\varepsilon_{su}\xi - \frac{7}{8}(1-\xi)}{\varepsilon_{su}} \right) dbf_{cd}$	$F_c = \frac{5}{4}\xi dbf_{cd}$
$z_c = \left(1 - \frac{1}{3}\xi\right)d$	$z_c = \left\{1 - \frac{\varepsilon_{su}\left[\frac{1}{2}\xi^2 - \frac{7}{8}\xi\frac{(1-\xi)}{\varepsilon_{su}} + \frac{49}{384}\frac{(1-\xi)^2}{\varepsilon_{su^2}}\right]}{\varepsilon_{su}\xi - \frac{7}{8}(1-\xi)}\right\}d$	$z_c = \left(1 - \frac{7}{30}\xi^2\right)d$
$M_{Rd,1a} = F_c z_c$	$M_{Rd.1b} = F_c z_c$	$M_{Rd,2} = F_c z_c$

Table 7. Cross-sectional parameters derived using bi-linear model for normal strength concrete

Domain 1a
$F_c = \frac{1}{2}\xi dbf_{cd}$
$z_c = d\left(1 - \frac{1}{3}\xi\right)$
$M_{Rd,1a} = \frac{1}{2}\xi\left(1 - \frac{1}{3}\xi\right)bd^2 f_{cd}$
Domain 1b
$F_c = f_{cd}bd\left(\frac{\varepsilon_{su}\xi - \frac{1}{2}\varepsilon_{c3}(1-\xi)}{\varepsilon_{su}} \right)$
$z_c = \left[1 - \frac{\varepsilon_{su}\left[\frac{1}{2}\xi^2 - \frac{1}{2}\xi\frac{\varepsilon_{c3}(1-\xi)}{\varepsilon_{su}} + \frac{1}{6}\frac{\varepsilon_{c3}{}^2(1-\xi)^2}{\varepsilon_{su}{}^2}\right]}{\varepsilon_{su}\xi - \frac{1}{2}\varepsilon_{c3}(1-\xi)}\right]d$

$$M_{Rd,1b} = f_{cd} b d^2 \left[\frac{\left(\xi - \frac{1}{2}\xi^2 \right)(\varepsilon_{su} + \varepsilon_{c3}) - \frac{1}{2}\varepsilon_{c3} + \frac{1}{6}\frac{\varepsilon_{c3}{}^2 (1-\xi)^2}{\varepsilon_{su}}}{\varepsilon_{su}} \right]$$
Domain 2
$$F_c = f_{cd} b d \left[\frac{\xi(2\varepsilon_{cu3} - \varepsilon_{c3})}{2\varepsilon_{cu3}} \right]$$
$$z_c = \left[1 - \frac{2\varepsilon_{cu3}\xi\left[\frac{1}{2} - \frac{1}{2}\frac{\varepsilon_{c3}}{\varepsilon_{cu3}} + \frac{1}{6}\frac{\varepsilon_{c3}{}^2}{\varepsilon_{cu3}{}^2} \right]}{(2\varepsilon_{cu3} - \varepsilon_{c3})} \right] d$$
$$M_{Rd,2} = f_{cd} b d^2 \left\{ \frac{\xi\left(2\varepsilon_{cu3} + \varepsilon_{c3} - 2\varepsilon_{cu3}\xi^2 \left[\frac{1}{2} - \frac{1}{2}\frac{\varepsilon_{c3}}{\varepsilon_{cu3}} + \frac{1}{6}\frac{\varepsilon_{c3}{}^2}{\varepsilon_{cu3}{}^2} \right] \right)}{2\varepsilon_{cu3}} \right\}$$

Design procedures and example of calculation

For flexural members with rectangular cross-section loaded by design bending moment M_{Ed} dimensioning of area of flexural reinforcement may be performed using an algorithm presented in Table 8. As a first step we should assume concrete class and data for reinforcing steel, diameter of bars and then evaluate nominal thickness of the concrete cover.

Table 8. Design procedures for rectangular cross-section of flexural R/C member

1	Input data	$M_{Ed,}$ b, d, b, $f_{cd,}$ ε_{uk}
2	Calculation of strain limits	$\varepsilon_u = f_{yd} / E_s$ and $\varepsilon_{ud} = 0{,}9 \cdot \varepsilon_{uk}$
3	Calculation of tabular coefficient	$\mu = \frac{M_{Ed}}{f_{cd} b d^2}$
4	Coefficients read from specified tables	ω or ζ and ξ
5	Steel strain calculations (or read from tables)	$\varepsilon_s = \varepsilon_c \frac{(1-\xi)}{\xi}$

6	Steel stress calculations (or read from tables)	$\sigma_s = f_{yd} + \frac{f_{yd}(k-1)\left(\varepsilon_s - \frac{f_{yd}}{E_s}\right)}{\left(\varepsilon_{ud} - \frac{f_{yd}}{E_s}\right)}$
7	Area of flexural reinforcement calculations	$A_{s1} = \omega db \frac{f_{cd}}{\sigma_s}$ or $A_{s1} = \frac{M_{Ed}}{\zeta d \sigma_s}$
8	Checking the minimum area of reinforcement	$A_{s1,prov} \geq 0{,}26 \frac{f_{ctm}}{f_{yk}} bd$ $A_{s1,prov} \geq 0{,}0013 bd$

Calculation example of flexural reinforcement area for flexural beam
The beam with rectangular cross-section made of HSC class C70/85.
Data: b = 0,25m, h = 0,35m, a_1=0,04m, concrete class C70/85, steel BSt500SB.
The calculations are made for three values of bending moments: M_{Ed} = 25 kNm, 50kNm and 100 kNm using different models of cross-sectional analysis.

$$f_{cd} = \lambda \frac{f_{ck}}{\gamma_{c,HSC}} = 0{,}95 \frac{70}{1{,}56} = 43{,}6\,\mathrm{MPa}$$

The results of calculations are presented in Table 9.

Table 9. Results of calculations of steel area for rectangular cross-section made of HSC

Model for concrete	Horizontal top branch		Inclined top branch	
	Parabola rectangle	Bi-linear	Parabola rectangle	Bi-linear
M_{Ed}	**M_{Ed} = 25kNm;**			
F_c/bdf_{cd}	0,025	0,025	0,025	0,025
z_c/d	0,969	0,983	0,984	0,985
A_{s1} [cm^2]	1,94	1,89	1,71	1,78
M_{Ed}	**M_{Ed} = 50kNm,**			
F_c/bdf_{cd}	0,050	0,050	0,050	0,049
z_c/d	0,956	0,967	0,972	0,973
A_{s1} [cm^2]	3,88	3,83	3,57	3,56
M_{Ed}	**M_{Ed} = 100kNm**			
F_c/bdf_{cd}	0,102	0,101	0,101	0,101
z_c/d	0,936	0,942	0,943	0,944
A_{s1} [cm^2]	7,92	7,85	7,63	7,81

Summary and conclusions

The presented above analysis was performed having considered more effective way to use realistic properties of ductility of flexural steel based on a basic assumptions described in Eurocode 2 and also in Model Code 2010. In the design procedures of flexural members with the use of steel stress-strain relationship with inclined top branch it could be reached saving of flexural steel areas. As it can be shown in the examples of design of flexural beam the influence of more realistic model on the required area of flexural steel depends on values of bending moment. The saving of steel area is larger for smaller depth of neutral axis zone. The results are also influenced on ductility of flexural steel. The more beneficial savings of required steel area can be reached for reinforcing steel with larger ductility ratio $k = f_{tk} / f_{yk}$ using strain model with inclined top branch.

In the examples there were used flexural steel with a ductility factor k = 1,10. For this case maximum savings of the reinforcing steel reaches to about 10%. Using steel class C *(k* >1,15*)* reduction of required reinforcement area would be up to about 15%. However for for steel class C with *k* =1,35 we can expect reinforcing steel reduction up to about 35%.

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